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MISCONCEPTIONS, ATTITUDES AND TEACHER PREPARATION

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1. INTRODUCTION

The problem of students' low achievement in mathematics has been extensively investigated with respect to a variety of possible interacting variables including emotional. Mathematical learning may differ from learning in other subjects and probably requires special student and teacher efforts due to its level of symbolic language, abstraction and the hierarchical development of logical schemata. The still prevailing academic way of presenting new concepts, the frequently behavioristic automation in learning and applying procedures, the failure (on the part of teacher) to connect mathematics with real life situations, in short, the poor and unimaginative teaching has created and preserved social stereotypes, beliefs, fears, negative attitudes and anxiety which constitute major obstacles.

Among the various measures of attitudes widely used were the "weighted item" scales (Dutton, 1962) and the Likert-type scales-employed at the 1st IEA (Husen, 1967). Gradually the emphasis has been removed from "attitudes" to "anxiety", "predispositions" and finally to "misconceptions". And although the change is not only a matter of new terminology, it may be reasonably argued that there is much overlapping between them, or at least a common core. For instance the statement "in mathematics there is always a rule to follow in solving problems", which was an attitude item in the 1st IEA, could also be a myth such as "there is one best way to do a maths problem" (Frank, 1990), or one "misconception" (Borasi, 1990). Similarly an attitude statement such as "there is no place for originality in mathematics", (Husen, 1967, Hirschhorn, 1993), is essentially identical to the common misconception that "it is no good trying to reason things out on your own", (Borasi, 1990), or to the belief that doing "mathematics means following rules", (Frank, 1988).

Research has been recently concentrated on students' conceptions (or misconceptions) about specific mathematical topics, procedures or concepts such as multiplication and division (Graeber & Campbell, 1993), the derivative and other calculus concepts (Amit & Vinner, 1990), violation of legal mathematical rules (Blando, Kelley, Schneider & Sleeman, 1989), the general teaching-learning process (Frank, 1988) etc. Considering the question from a rather general point of view, Borasi (1990) claims that "dualistic" beliefs, prevailing about mathematics, may prove dysfunctional for their holders, as they may "cause expectations and behaviors leading to anxiety and academic failure", (p. 177). She also proposes a classification of students' misconceptions in four broad categories. They concern:

- The scope of mathematical activity: "To provide answers to well defined problems".
- The nature of mathematical knowledge: "Everything is either right or wrong, there is no room for personal judgement".
- The nature of mathematical activity: "To recall and apply rules and procedures to solve given problems".
- The origin of mathematical knowledge: "Mathematics is a finished product transmitted, possibly with some new additions, from one generation to the other"[1].

In a rather similar way Paulos (1992) refers to the following five "crippling and widely held misconceptions": Mathematics is computation, Mathematics is a rigidly hierarchical subject, Mathematics and narrative are disparate activities, Mathematics is only for the few, Mathematics is "numbing". Frank (1988) analysing observational and interview results of mathematically talented middle school students concludes that the major misconceptions are that: Mathematics is computation, problems should be solvable in just a few steps, the goal of mathematics is to obtain right answers, the role of the student is receptive-demonstrative while the teacher's role is to transmit and verify (by testing). Despite all existing literature, Schoenfeld (1985) claims that "typical instruction and testing provide little opportunity for students to demonstrate the breadth and depth of their misconceptions" (p. 13).

Concerning the origination and development of such negative beliefs and counterproductive conceptions, apart from the existing social stereotypes, the environment and the stages of intellectual development, (Borasi, 1990), a major role is also played by the existing relationship between the learner and the specific body of knowledge (Charlot, 1982). This relationship is a synthesis of all the positive and negative experiences of the individual with mathematical knowledge, the places and conditions under which it has been developed, individual expectations and professional perspectives, the self-image and certainly the mediator, between mathematics and the student, person. The role of the teacher in the process of shaping and reshaping of this relationship is crucial as she is almost entirely responsible for the total student mathematical experiences. But the organization of meaningful mathematical activities depends heavily upon the teacher's own relationship to the subject (Hyde, 1989). Hence the key step in any effort to improve students' conceptions is the development of "productive" teacher conceptions about mathematics.

During the period 1989-1992 a research program was undertaken by the University of the Aegean aiming at the assessment of primary and secondary school-leavers' mathematics performance and attitudes and of the prospective teachers' attitudes. At the same time a mathematics content course was designed based on the historical development of concepts and mathematical ideas aiming at changing, in the positive direction, students' motivation, knowledge and beliefs about mathematics. Selected findings of this program will be presented and discussed in this paper, with emphasis on the prospective teachers' attitudes and preparation.

2. THE PROCEDURE

The above mentioned program consisted of three related surveys: The Primary School Survey (the questionnaire was given in June 1989), the Secondary School Survey (the questionnaire was given in June 1990), and the Prospective Teacher Attitude Survey, which took place in the succeeding year (late 1991 early 1992).

2.1 The Samples. The subjects of the Primary Study were 980 school-leavers from the island of Rhodes and one district of Cyprus (28% and 20% of the total population respectively), selected by the stratified sampling technique. The subjects of the second study were 1195, 9th grade students from Rhodes and three districts of Cyprus, (around 20% of each district population). The subjects of the Prospective Teachers Attitude Survey were 385 prospective primary and kindergarten teachers from three rural-area Universities: Aegean, Thessalias and Florinas (only primary). No randomization was attempted in this study but the proportion of participants was in all cases well above 30% of the total population. Some significant characteristics of this sample were the following:

- Girls outnumbered boys by far, 327 and 58 respectively.
- 228 were prospective elementary teachers and 157 kindergarten teachers.
- The vast majority, 80%, opted not to take mathematics in the entrance exams.
- Three students out of four were coming from a non-mathematics section of the high school.
- Students come mostly from low socio-cultural families, 183 had fathers and 233 mothers with only elementary school education.

2.2 The Scales. The same attitude scales were used for both the Primary and the Secondary Surveys. The scales consisted of sixteen Likert-type items measuring attitudes along four dimensions.: Mathematics as a process, the significance of Mathematics in society, the difficulty of learning Mathematics, and the enjoyment derived from activities with Mathematics. For the prospective teachers' attitudes four related and complementary scales were used:

- The Dutton Scale, slightly adapted, included eighteen items weighted from 1.0 to 10.5
- The "justification scales". Two ten-item scales one for liking and one for disliking mathematics (the statements were similar to those found by Smith[2], 1964).

- The "comparison" scale. The students were required to choose among the basic school courses the one they enjoyed best, considered most useful, most difficult and the one they would like to teach.
- The "self-evaluation" scale, where they were expected to locate themselves on a number line from 1 to 11, the first indicating extremely negative and the second extremely positive attitudes.

3. THE RESULTS

3.1 At both the Primary and the Secondary Surveys the subjects were found to hold rather poor attitudes toward "mathematics as a process", the average value being near 9 out of 16 (4x4) maximum, where 8 naturally represents neutral attitudes. Table 1 shows the mean value by study, by area and by scale dimension. Two conclusions seem to be justified on the basis of this table.

- The "process" dimension of the scales is the poorest in terms of students' attitudes, whereas the "significance" is the best out of the four.
- The whole picture is getting worse from the Primary to the Secondary Study, the figures in all cells found smaller. The greatest difference was found at the "enjoyment" dimension where the observed differences approach two units, and they are statistically significant, ($p<0.01$), by the t-test.

Table 1. The Mean and Standard Deviation on the Attitude Scales by Study, by Area and by Scale Dimension

Study	Primary Study				Secondary Study			
	Limassol		Rhodes		Cyprus		Rhodes	
Dimension	$\bar{X}$	σ	$\bar{X}$	σ	$\bar{X}$	σ	$\bar{X}$	σ
process	9.5	2.22	9.1	2.34	9.2	2.21	8.9	2.16
difficulty	12.0	2.48	11.7	2.30	11.8	2.30	11.8	2.39
significance	12.2	2.77	12.5	2.71	12.0	2.74	11.5	2.48
enjoyment	12.2	2.53	11.9	3.19	10.8	3.60	9.9	3.40

The two statements with most negative attitudes ($\bar{X} < 2$ - the neutral value) were the following:

- **There is very little room for originality in mathematics.**
- **In mathematics, if one misses a few classes, it is very difficult to catch up.**

3.2 The Prospective Teachers'Attitudes. The results of this study are presented in details and discussed in some depth, separately for the Dutton, justification, comparison and the self-evaluation scales.

Dutton Scale. The responses of the subjects on each item of the Dutton's scale distinctly for elementary and kindergarten teachers are summarized in Table 2, where the respective frequencies found by Smith (1964) are also included. On the basis of this evidence the following comments seem well justifiable.

- There is an alarmingly high proportion of participants with extremely negative attitudes - 18% and 26% respectively "detest mathematics and avoid them always"! The situation is certainly not balanced by the relatively high percentages on the upper part of the scale.

Table 2. Responses on the Duttons's Attitude Scale and the Results of Smith's Study

Scale Value	Statement Number	Attitude Statement	Element (231) %	Kind. (154) %	Smith (123) %
1.0	13	I detest mathematics and avoid using them all the times	18	26	0
1.5	18	I have never liked mathematics	9	16	11
2.0	11	I am afraid of doing word problems	9	16	46
2.5	16	I have always been afraid of Maths	23	18	24
3.3	9	Mathematics is something you have to do even though it is not enjoyable	40	35	24
3.7	2	I don't feel sure for myself in Mathematics	46	56	41
4.6	6	I don't think mathematics is fun, but I always want to do well in them	51	55	26
5.3	7	I am not enthusiastic about maths, but I have no real dislike for them	50	52	28
5.6	4	I like mathematics, but I like other subjects just as well	49	29	54
5.9	8	Mathematics is as important as any other subject	43	41	76
6.7	14	I enjoy doing problems, when I know how to do them	51	50	64
7.0	10	Sometimes I enjoy the challenge presented by a mathematical problem	47	26	62
7.7	5	I like mathematics because it's practical	29	18	40
8.6	3	I enjoy seeing how rapidly and accurately I can work mathematical problems	32	24	41
9.0	12	I would like to spend more time in school working mathematics	28	21	21

9.5	1	I enjoy working on and thinking about mathematical problems outside of school	29	15	34
9.8	17	I never get tired of working with mathematics	15	8	15
10.5	15	Mathematics thrill me; I like it better than any other subject	8	5	0

- The highest percentages of agreement, (near 50%), are observed around the middle of the scale-weighting factors between 3.7 and 7.0 - which means on the rather neutral statements.
- Primary teachers responded more positively than kindergarten teachers i.e., the agreement frequencies were smaller on the negative statements and greater on the positive statements.
- Special note deserves the fact that 46% of the prospective primary and 56% of the kindergarten teachers state that they feel no confidence in mathematics. What chance do they have to teach them successfully?
- There are certainly a lot of differences between the results of this study and that by Smith (1964). We are however in no position to decide to which degree they are due to some adaptation of the statements, the cultural differences, or the historical changes.

Appendix A illustrates the superiority of the prospective elementary over the kindergarten teachers, with respect to this scale. Similarly those who sat mathematics in the entrance exams are seen to have more favorable attitudes than those who did not.

The "Justification" Scales. Results are summarized in Table 3, separately for each of the three universities and each of the two majors (elementary and kindergarten). The most popular reasons-explanations of students' liking of mathematics are seen to be indicated in items 7,2,3, and 9, i.e. preservice teachers like mathematics as it exercises the mind, it's necessary for living, useful and practical, and it is logical and coherent.

Table 3. Responses on the "Justification Scales" by University and Major.

University Scale item	Rhodes (231) %	Thessalia (88) %	Florina (66) %	Total Element. (228)	Total Kinder. (147)
Reasons for liking maths					
1. Interesting and challenging	28.6	26.1	34.8	31.6	23.2
2. Necessary in modern life	54.1	51.1	48.5	53.5	53.7
3. Practical and useful	46.3	42.2	40.1	44.7	44.5
4. I can understand them	12.1	9.1	19.7	15.4	8.4
5. Gives a feeling of success	23.4	23.9	28.8	26.3	21.3
6. It's fun	10.8	17.0	12.1	12.7	10.3
7. It excercises the mind	60.2	63.6	63.6	62.7	59.4
8. Where I had good teachers	17.3	13.6	16.7	17.5	14.2
9. Logical and coherent	36.4	36.4	37.9	39.0	32.9
10. Rewarding	21.2	28.4	30.3	26.8	20.6
Reasons for disliking maths					
1. I don't understand them	29.0	34.1	25.8	27.2	31.6
2. Can't solve the problems	33.8	34.1	24.2	29.4	35.5
3. Never had done well	35.5	39.8	25.8	29.8	39.4
4. Poor teaching	33.3	35.2	24.2	28.9	35.8
5. Lack of teacher enthusiasm	36.8	34.1	33.3	32.9	39.4
6. Never related to practice	10.8	14.8	15.1	10.1	14.2
7. Require too much thinking	20.1	20.5	16.7	16.2	24.5
8. Takes too much time	19.0	17.0	15.1	15.4	21.9
9. Afraid of them	43.7	40.1	42.2	40.8	46.5
10. Exercises used as punishment	12.6	17.0	16.7	12.7	15.5

The lowest proportions are found on the enjoyment item, followed by the understanding and next on having good teaching. The students don't like mathematics because they are "afraid of them", "lack of teacher enthusiasm" and they "never did well". On the contrary they don't seem to agree that they are "never related to practical situations" neither they consider "exercises as punishment".

The Comparison Scale required the student to choose his favourite subject among Greek, Mathematics, Physics, Chemistry, History or other, with respect to four factors: enjoyment, usefulness, difficulty, and willingness to teach. The responses of the participants are shown on Table 4. Only one out of ten students enjoyed best doing mathematics during the secondary schooling, and quite naturally, the same proportion of students stated that they were looking forward to teaching them, although the subject was considered to be the most useful by a higher proportion, 19%, being second only to Greek. Mathematics is ranked second in difficulty, 32%, just one unit under Chemistry. The most popular subject with respect to all four dimensions is the Greek language and History is the second on the "enjoyment" and the "wishing to teach".

Table 4. Responses on the Comparison Scale by University

	Greek N	Greek %	Mathematics N	Mathematics %	Physics	Chemistry	History	Other
Rhodes								
Enjoyment	106	46	22	10	12	5	65	21
Usefulness	119	52	51	22	10	3	30	18
Difficulty	6	3	75	32	54	80	2	15
To Teach	99	43	27	12	4	4	68	27
Thessalia								
Enjoyment	39	44	10	11	4	18	18	9
Usefulness	41	47	13	15	7	4	17	7
Difficulty	4	5	34	39	20	23	5	2
To Teach	37	42	7	8	4	5	19	16
Florina								
Enjoyment	33	50	5	8	4	2	19	3
Usefulness	36	54	9	14	6	1	11	3
Difficulty	3	4	15	23	21	25	0	2
To Teach	30	46	6	9	1	1	24	4
Total								
Enjoyment	178	46	37	10	20	15	102	33
Usefulness	196	51	72	19	23	8	58	28
Difficulty	13	3	123	32	95	128	7	19
To Teach	166	43	40	10	9	10	111	48

The differences in the proportions between the three universities are smaller than one might have expected. In fact they seem quite consistent.

Appendix B illustrates the comparison on the the Dutton's scale by yeargroup (1+2 vs. 3+4), and the results on the comparison scale. It seems that the upper group students have better attitudes, though the trend is not really conclusive.

Appendix C illustrates the responses on the liking part of the justification scale by major, by yeargroup and according to whether they sat mathematics or not. A clear superiority of elementary over kindergarten and of those who sat over those who didn't is evident, whereas the upper years students seem to perform better than the lower. The first two findings seem to be in agreement with the results on the second part of the scale, the reasons for disliking mathematics as shown in **Appendix D**. In some items of this part of the scale the proportions of the two upper years students are bigger and in other they are smaller than those of the first two years; hence no definite decision could be reached concerning the comparison according to yeargroup.

The "Self-Evaluation" scale provided a direct means to the subjects for stating their own relationship to mathematics. Though the linearity of the scale could be an oversimplification of this relationship, evidence from this measure could be used as supplementary. The mean values for the various groups of the sample were found near six, the neutral value while two of them were under six, see Table 5.

Table 5. The Responses on the Self-Evaluation Scale: Mean and Standard Deviation by University Major and Sex.

	Rhodes				Thessalia		Florina
	Elem.	Kind.	Boys	Girls	Element.	Kinder.	Elementary
N	122	108	36	195	40	47	66
$\bar{X}$	6.49	5.82	5.42	6.32	6.32	6.09	6.83
σ	2.62	2.53	3.12	2.46	2.62	2.58	2.40

It is clear again that prospective elementary teachers have superior self-esteem with respect to mathematics than kindergarten. Similarly girls seem to be better off than boys with respect to their self-image about mathematics. No further investigation of this finding was attempted due to the limited number of boys in the sample.

The three charts in Appendix E illustrate the responses on the self-evaluation scale by major, by the factor "sat mathematics" or not, and by yeargroup. The prospective elementary teachers have significantly better attitudes than the kindergarten ($p<0.01$, by the t-test), those who sat mathematics have significantly better attitudes than those who did not ($p<0.01$), and the upper group students were found to have not significantly better attitudes ($\bar{X}_1 = 6.51$, $\bar{X}_2 = 6.12$, $p=0.21$); so, exposure to the system was not found to have significantly improved students' attitudes towards mathematics.

4. DISCUSSION AND IMPLICATIONS FOR TEACHER EDUCATION

From the two Primary and Secondary Studies it was found that students in Rhodes and Cyprus have poor attitudes towards mathematics, which they develop in the negative direction as they grow older. Negative attitudes were found to be related to anxiety, low performance and mathematics avoidance (Hembree, 1990). The prevailing views that a wider, more relevant, and challenging curriculum can, under normal conditions, develop mathematical thought and understanding of all students, (Womack, 1988), has one crucial prerequisite: that it is appropriately designed and taught by well prepared teachers. Yet the majority of the prospective teachers participated in this study had already avoided to take mathematics in high school, they (naturally) didn't sit mathematics in the entrance exams, most of them have negative attitudes toward mathematics, lack of self-confidence (they don't want to teach the subject) and generally have poor personal relation to it. Yet, according to the educational system, all of them will have to teach mathematics to young children at a very sensitive age, when they become teachers.

Based on the findings from the Prospective Teachers' Attitude Study the following conclusions need to be emphasized.

- Success in mathematics brings more success and failure is associated with mathematics avoidance and negative attitudes. The responses on the "justification" scale, both the liking and the disliking parts are quite clear about this.

- The significant attitude superiority of those who sat mathematics over those who didn't, may be interpreted as an indication that mathematics avoidance and poor attitudes go together.
- Among the reasons for disliking mathematics, "poor and non enthusiastic teaching" is ranked by prospective teachers on the top.

It seems that the system, concerning the teaching and learning of mathematics, is to a large degree moving into a vicious circle. Teachers without love or understanding of mathematics, unwillingly teach the subject, consequently developing poor learning, misconceptions and negative attitudes. Some of those students, certainly not the best in mathematics, choose to become teachers, probably due to lack of other choices, and the problem is steadily reproduced. In a recent study some teachers in Greece described the situation in very clear terms..."I had to teach grade 5 and I didn't know how to multiply fractions, because I was weak in mathematics in the secondary school" (Frederikou & Folerou, 1991, p.66), a desperate voice.

The big question is definitely how to break down this vicious circle, at which stage will this be easier or possible? Several solutions have been proposed so far, most of them quite reasonable, i.e. small group discussions, co-operative problem solving, self-assessment, flexible curriculum, process oriented teaching etc, and concerning the key-factor in any innovative attempt, the teacher, there is the in-service education plus the assistance by enrichment material. All these ideas are certainly good, but is also known that beliefs, fears, attitudes and misconceptions are resistant to change, particularly when they become convictions year by year. Therefore, a good break through point may be the pre-service period where the prospective teachers are exposed to long lasting experiences organized under the leadership of "experts" in the field of mathematics education. Dodd (1992), reports being a "maths-phobic" person, who believed that it was an incurable disease; she finally managed to get through by just attending, at adult age, one well designed course in statistics.

4.1 **The proposal**. In a recently designed program for preservice elementary teachers, two mathematics content courses and one method have being included plus the teaching practice. Of the two content courses the first one is designed to proceed along the developmental historical line. In the next pages some information about and analysis of this course will be given.

Why History of Mathematics? In the contemporary school the learners are expected to participate in the process of constructing their own mathematical ideas and discovering algorithms and procedures. The persons responsible for directing such learning should themselves have a real understanding of mathematics. Francis (1988) emphasizes the real fact that courses taken by prospective teachers may be the last opportunity for acquiring such an appreciation. He also claims that "one promising and meaningful provision for developing mathematical appreciation falls into the area of the history of mathematics" (p.82). The developmental nature, the growth of the core ideas and how they are interrelated are points emphasized by Dossey (1988), who also includes among the objectives of the mathematical studies of preservice elementary teachers the development of the ability "to describe the historical and cultural significance of some of the mathematical concepts and principles studied in the K-6 mathematics program", (p.26).

Concerning the cultural environment under study, history of mathematics has been tested by Philippou (1978), as a means of improving secondary students' attitudes and it worked. After the teaching of a unit from the history of mathematics the experimental group's attitude-change was found significantly better than the control group's.

Rational and brief description of the course. A considerable part of all mathematics in the K-6 program has its origins or was developed from the early historic period. Yet this evolution took centuries or millennia and it is still continuing. It is envisaged that a knowledge of this evolutionary process will make a lot of difference. For instance, understanding of the number system and the place value property would be best appreciated through a historical study of the number systems. The prospective teacher will be given the opportunity to understand that the place value is a powerful property as it makes the arithmetic operations easier, but there is nothing divine about the decimal system. He will appreciate the struggle of the Babylonians to do commercial calculations in the hexagesimal system, of the Egyptians to derive their multiplication algorithm and of Greeks to work in the alphabetic system. He will also realize that despite its strengths the decimal system is not generally accepted as the best. If mathematicians were to propose now the best possible system it is not at all certain that the decimal would be the one. For instance the electronic computer uses the binary system which has certain advantages (and disadvantages) over the decimal.

One misconception or common myth is that there is only one (best) solution to each problem. It is envisaged that after a study of the three famous problems, i.e. doubling the cube, trisecting the angle and squaring the circle, the prospective teacher would change his mind. He will be encountered with and hopefully discover, a lot of admirable "solutions" of unsolvable problems and hence realise gradually the significance of the initial conditions - the rules of the game. Agreement on the rules and a taste of contemporary mathematics will also be realized through the study of Euclid's axiomatic method. Where does the postulate system come from, what is so important about the fifth postulate, and what an abundance of equivalent propositions!

The course developed is a three credit semester course and it includes the discussion of topics such as:Ancient and modern number systems, Pythagorean mathematics (including the famous theorem and some of its generalizations, polygonal numbers and the discovery of irrationals), the three famous problems of antiquity, Euclid's Elements (the axiomatic method and selected theorems), a taste of Archimedean mathematics (helix and the trisection, the "method"), Ptolemy's theorem and the first trigonometric tables etc. The course is partly based on Eves (1980) "Great Moments in Mathematics".

Whether the course was successful in developing positive students' attitudes towards mathematics it is not yet known. The lower and upper years student attitudes comparison was inconclusive, as it was mentioned above. However, since according to the findings of other studies the attitudes are progressively getting worse, even for university students majoring in mathematics, Aksu[3] (1991), the non-decrease or even better the non-significant increase found in this study may be considered as encouraging. The project is currently continuing in Cyprus, where the author has moved, and will be soon re-assessed.

Notes

1. The second draft of this paper was already completed when I found out that Cooney et al (1993) have an almost identical quotation of Borasi's same four points. Their quotation, however, was meant in a rather different context.

2. Smith (1964) has found similar to those statements in students' responses to an open-ended question concerning the reasons for liking or disliking mathematics. In the present study the subjects were simply requested to tick only those of the items they agreed with.

3. Aksu (1991) investigated attitudes of college students majoring in mathematics and prosepctive secondary school mathematics teachers by administering an Aiken scale at the freshman year and for a second time after a 2-year time interval. The overall mean attitude score of the second administration was found significantly lower for the total group and the Department of Mathematics students, but not for the Department of Mathematics Education students.

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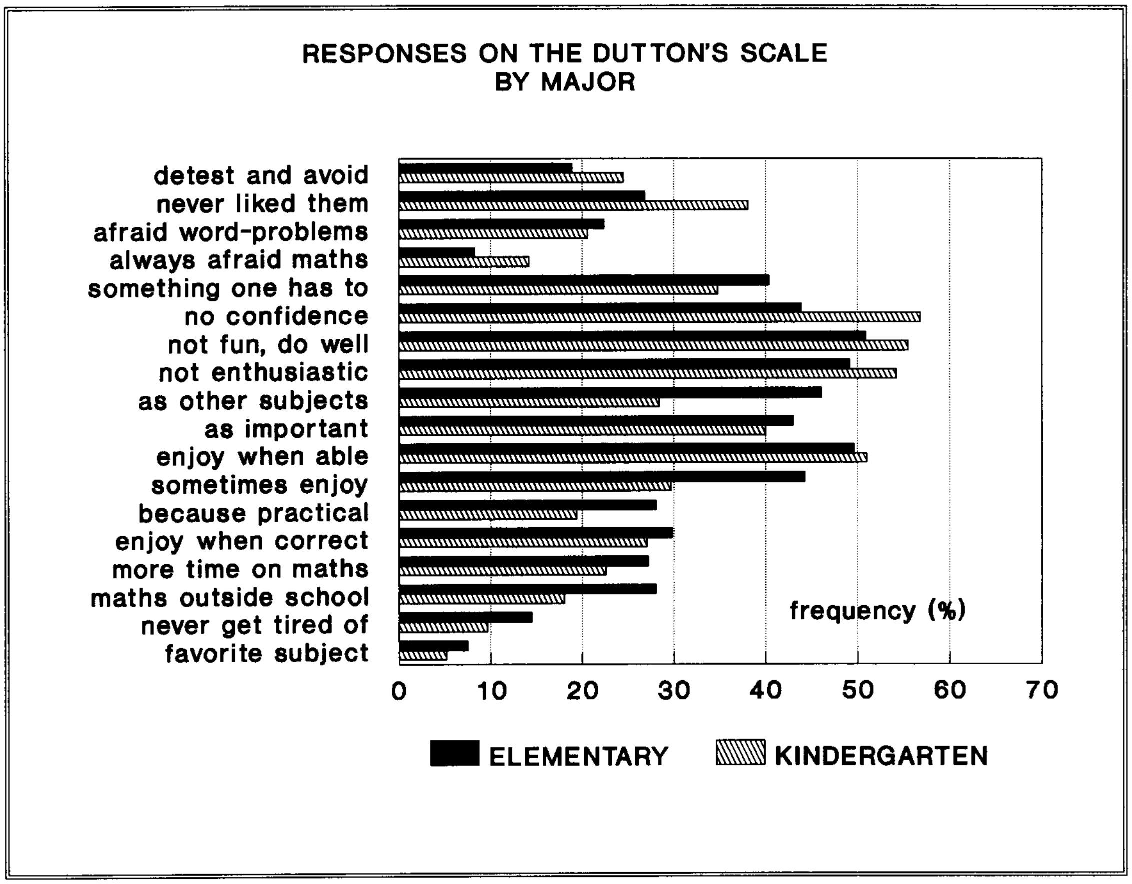
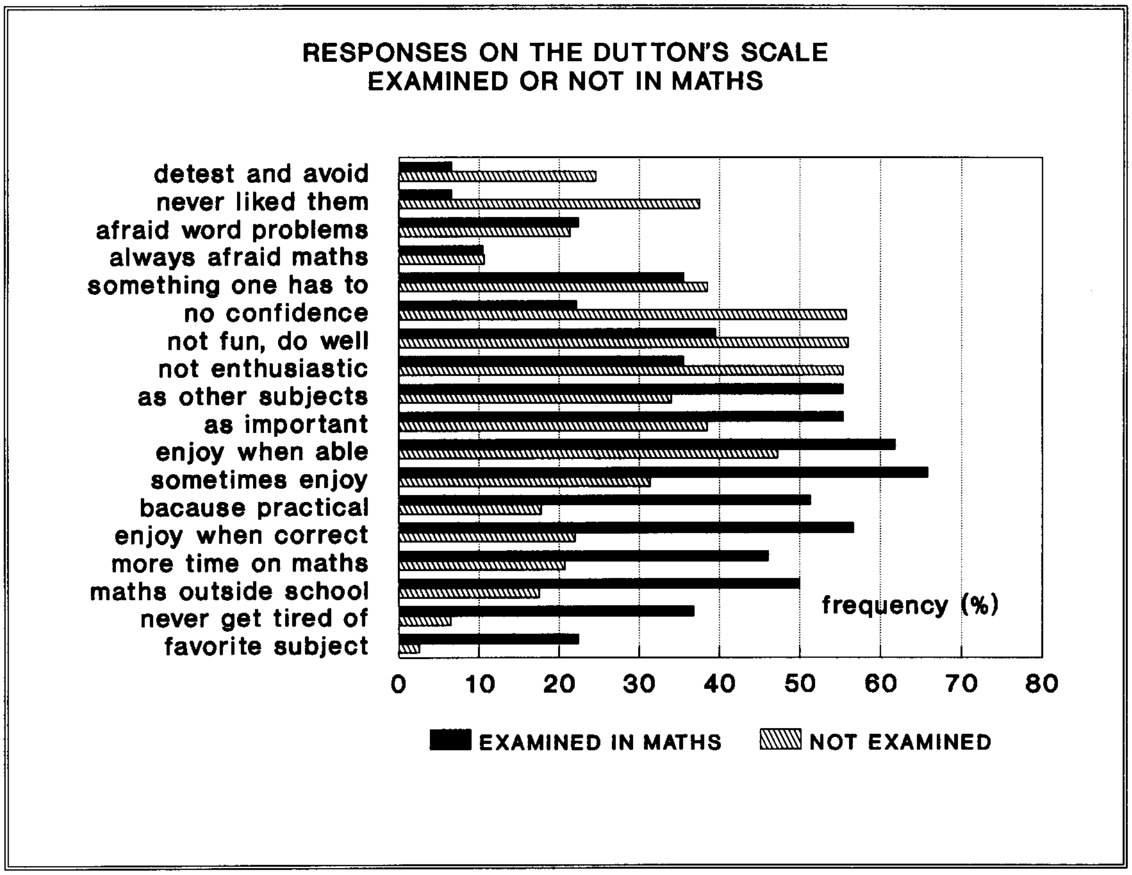
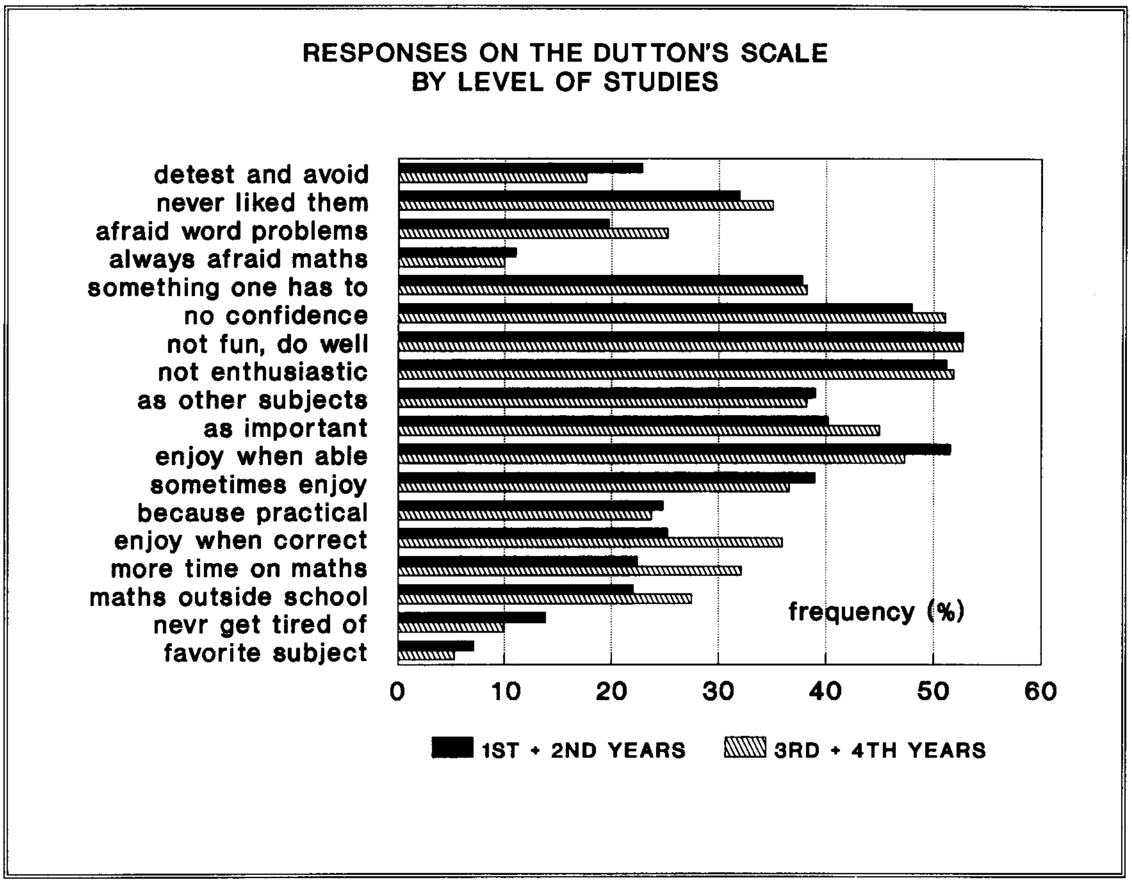
APPENDIX A


RESPONSES ON THE DUTTON'S SCALE
BY MAJOR
detest and avoid
never liked them
afraid word-problems
always afraid maths
something one has to
no confidence
not fun, do well
not enthusiastic
as other subjects
as important
enjoy when able
sometimes enjoy
because practical
enjoy when correct
more time on maths
maths outside school
never get tired of
favorite subject
frequency (%)
0
10
20
30
40
50
60
70
ELEMENTARY
KINDERGARTEN

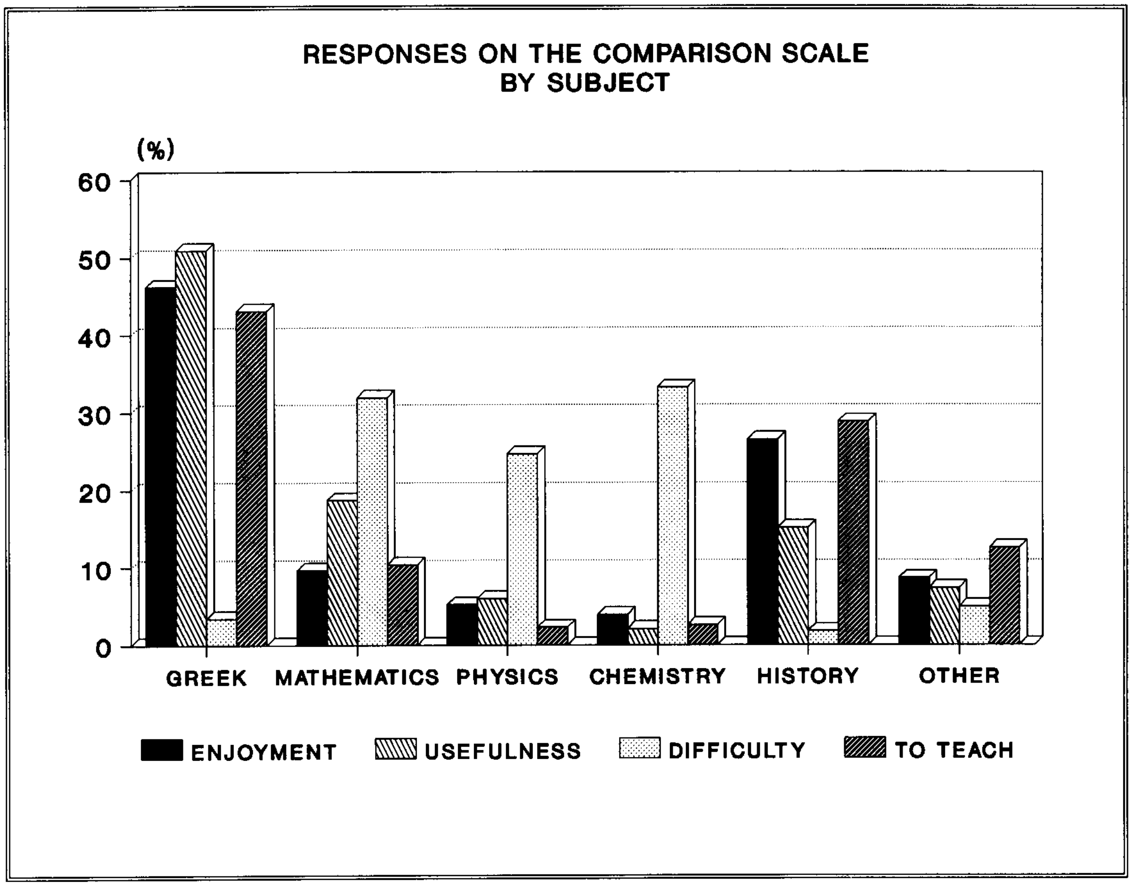


RESPONSES ON THE DUTTON'S SCALE
EXAMINED OR NOT IN MATHS
detest and avoid
never liked them
afraid word problems
always afraid maths
something one has to
no confidence
not fun, do well
not enthusiastic
as other subjects
as important
enjoy when able
sometimes enjoy
bacause practical
enjoy when correct
more time on maths
maths outside school
never get tired of
favorite subject
frequency (%)
0 10 20 30 40 50 60 70 80
EXAMINED IN MATHS
NOT EXAMINED

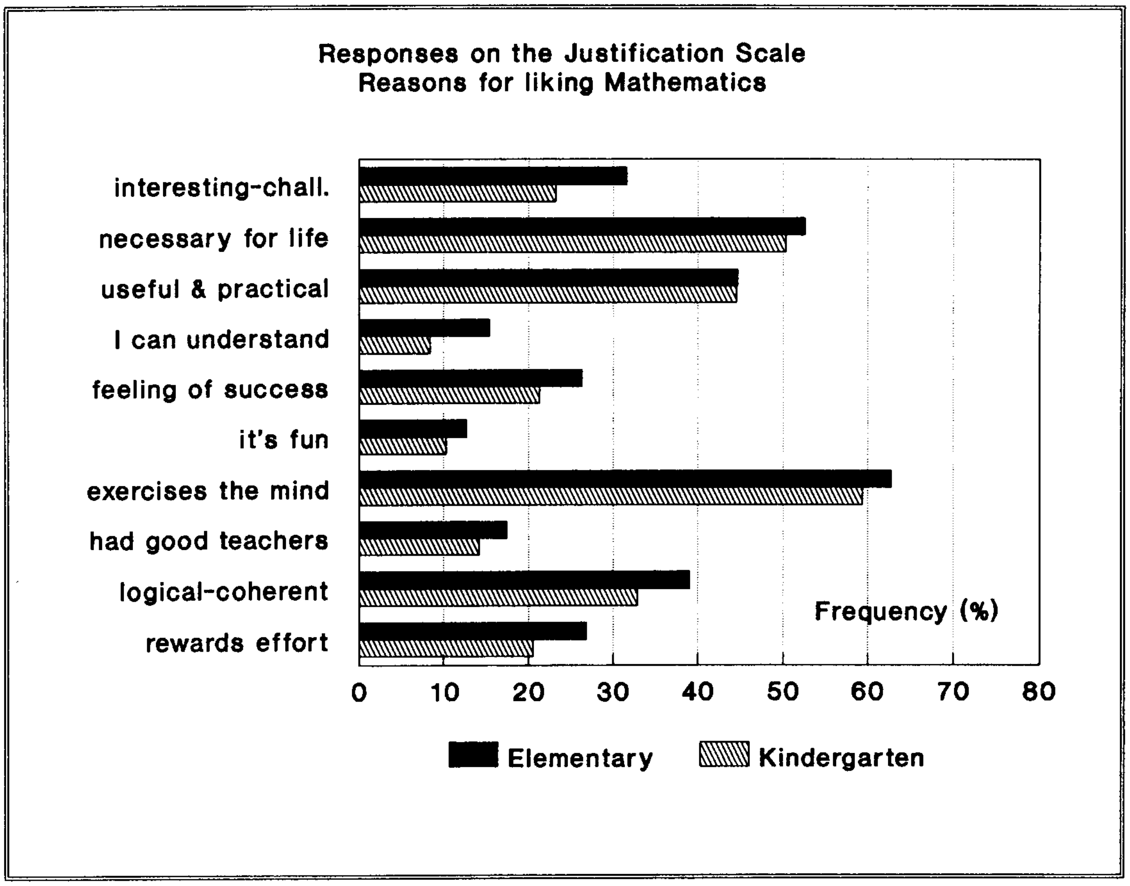
APPENDIX B



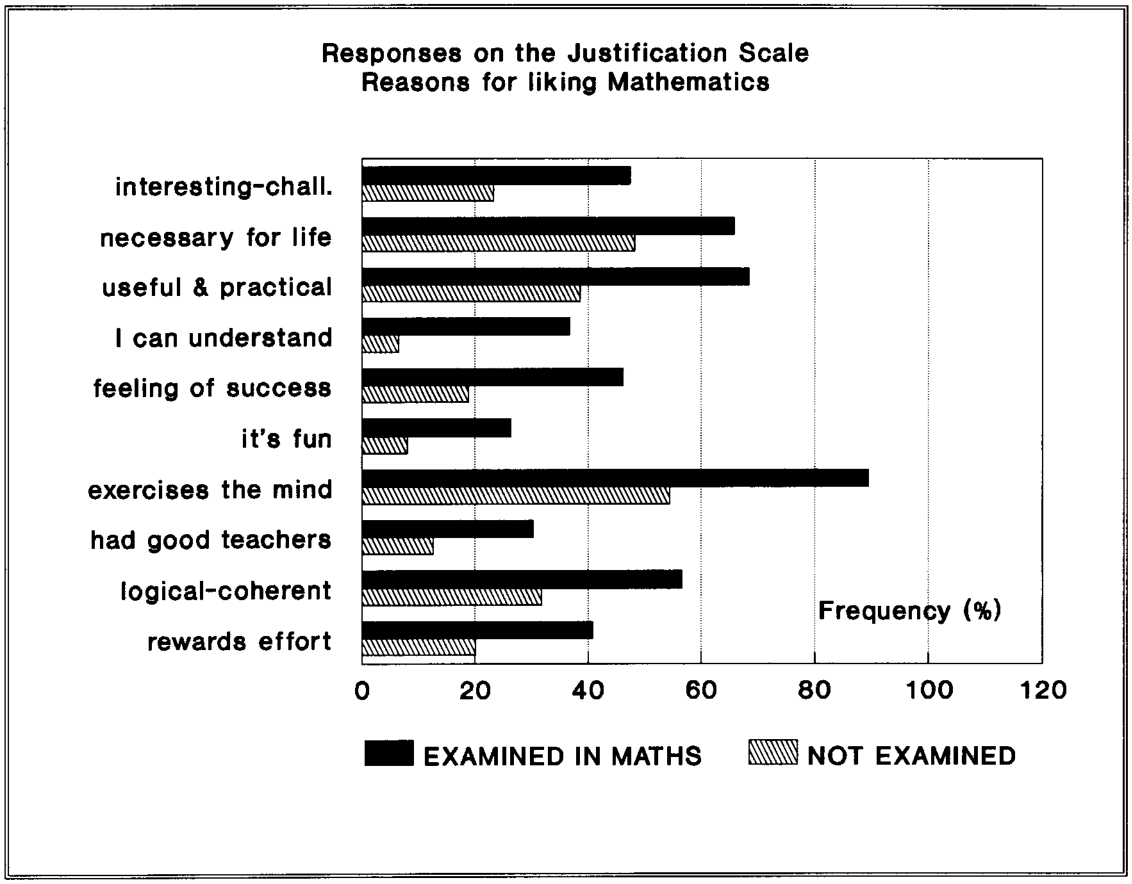
RESPONSES ON THE DUTTON'S SCALE
BY LEVEL OF STUDIES
detest and avoid
never liked them
afraid word problems
always afraid maths
something one has to
no confidence
not fun, do well
not enthusiastic
as other subjects
as important
enjoy when able
sometimes enjoy
because practical
enjoy when correct
more time on maths
maths outside school
nevr get tired of
favorite subject
frequency (%)
0
10
20
30
40
50
60
1ST + 2ND YEARS
3RD + 4TH YEARS

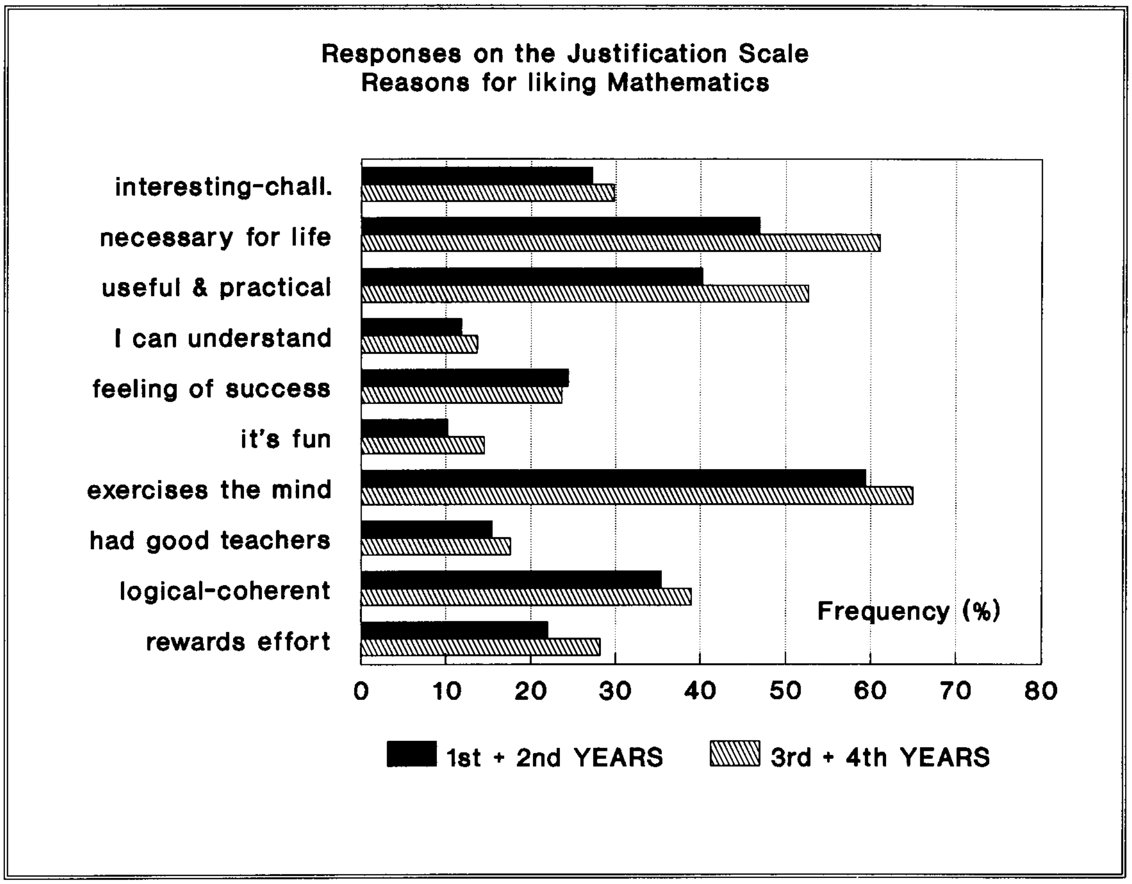

RESPONSES ON THE COMPARISON SCALE
BY SUBJECT
(%)
60
50
40
30
20
10
0
GREEK
MATHEMATICS
PHYSICS
CHEMISTRY
HISTORY
OTHER
ENJOYMENT
USEFULNESS
DIFFICULTY
TO TEACH

APPENDIX C



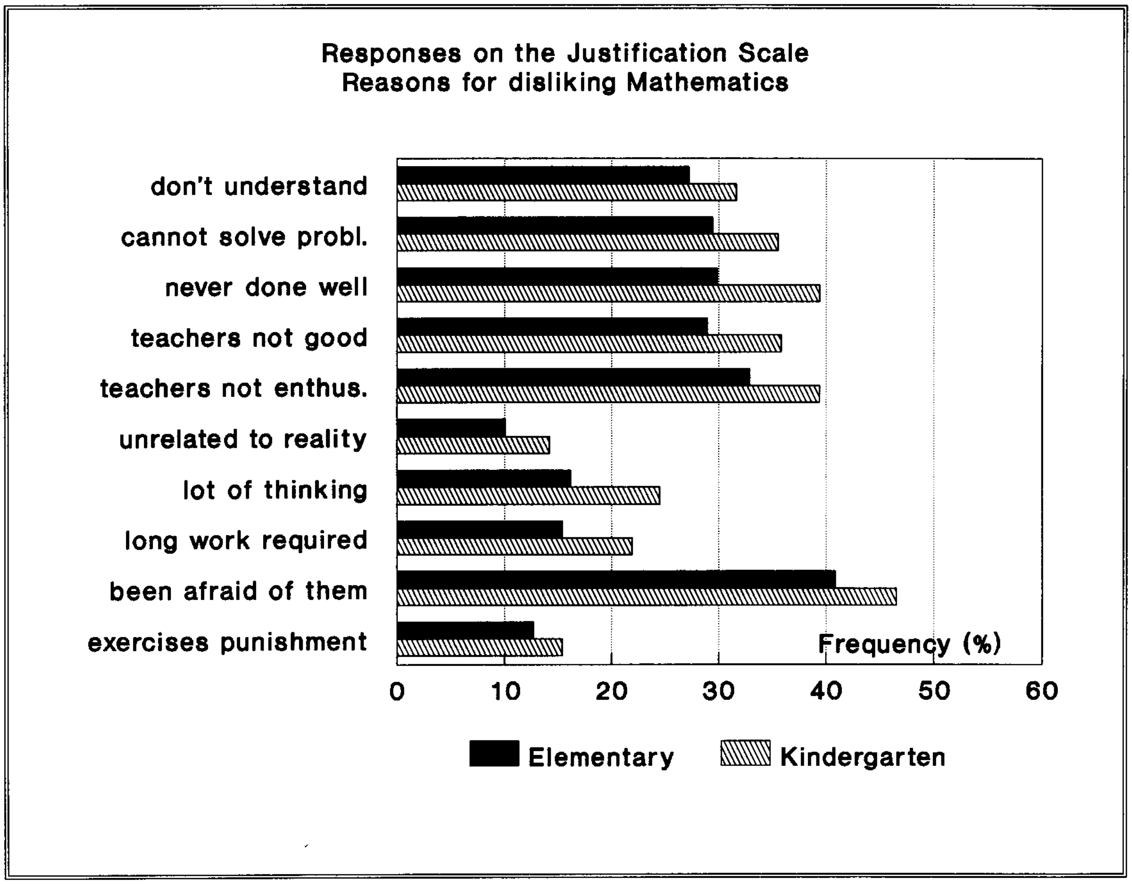
Responses on the Justification Scale
Reasons for liking Mathematics
interesting-chall.
necessary for life
useful & practical
I can understand
feeling of success
it's fun
exercises the mind
had good teachers
logical-coherent
rewards effort
Frequency (%)
0
10
20
30
40
50
60
70
80
Elementary
Kindergarten

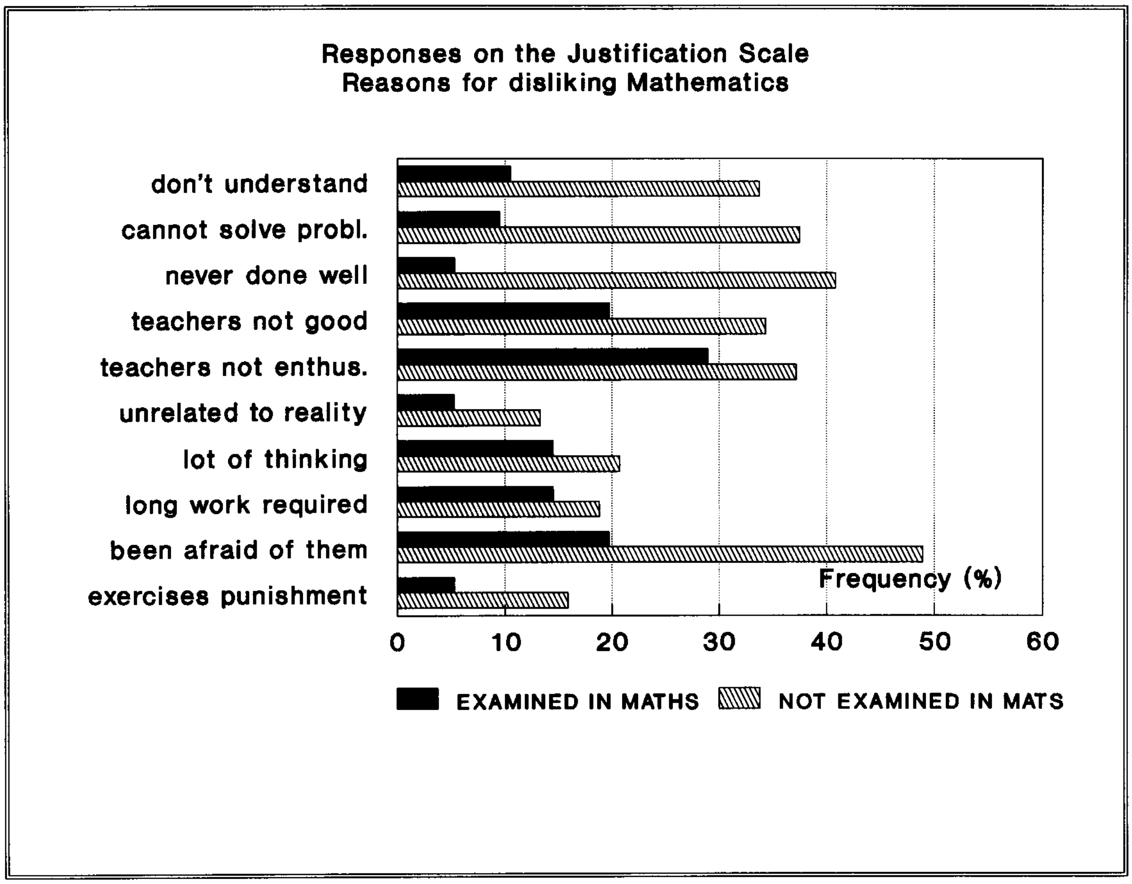

Responses on the Justification Scale
Reasons for liking Mathematics
interesting-chall.
necessary for life
useful & practical
I can understand
feeling of success
it's fun
exercises the mind
had good teachers
logical-coherent
rewards effort
Frequency (%)
0
20
40
60
80
100
120
EXAMINED IN MATHS
NOT EXAMINED

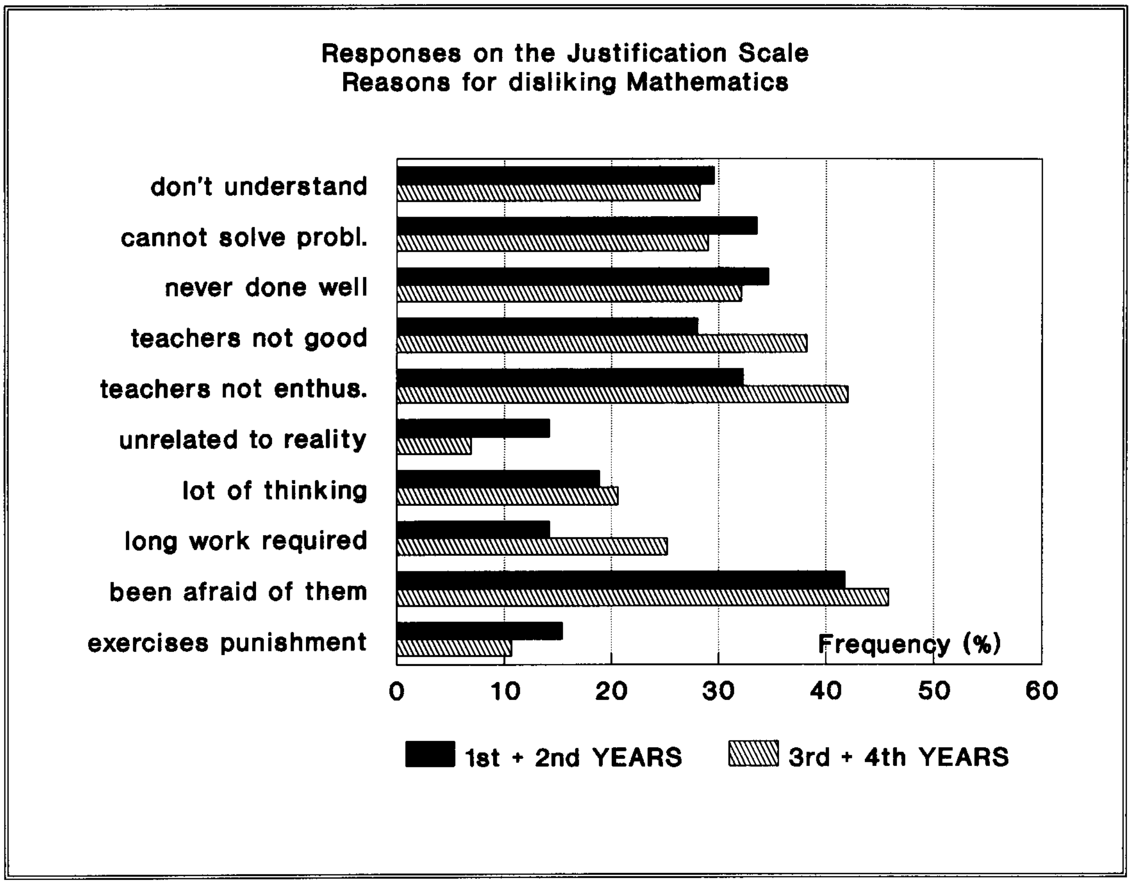


Responses on the Justification Scale
Reasons for liking Mathematics
interesting-chall.
necessary for life
useful & practical
I can understand
feeling of success
it's fun
exercises the mind
had good teachers
logical-coherent
rewards effort
Frequency (%)
0
10
20
30
40
50
60
70
80
1st + 2nd YEARS
3rd + 4th YEARS

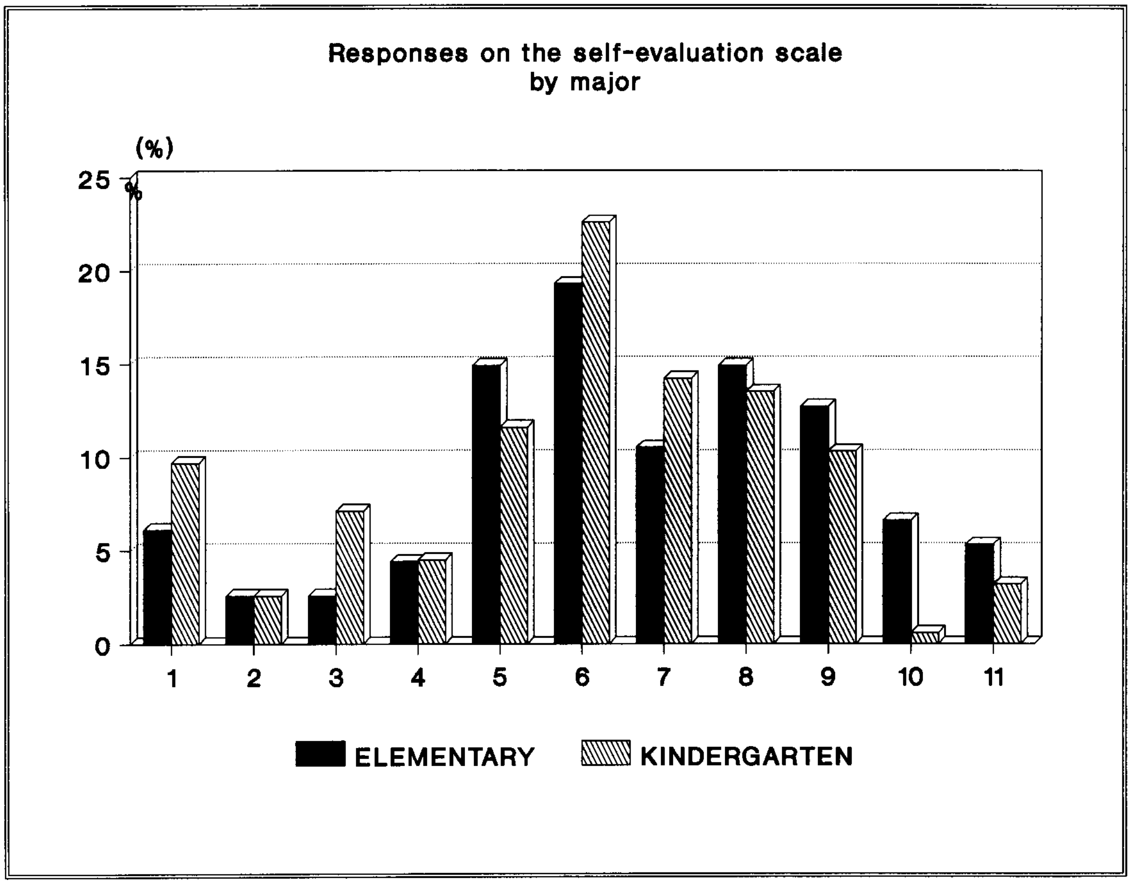
APPENDIX D


Responses on the Justification Scale
Reasons for disliking Mathematics
don't understand
cannot solve probl.
never done well
teachers not good
teachers not enthus.
unrelated to reality
lot of thinking
long work required
been afraid of them
exercises punishment
Frequency (%)
0
10
20
30
40
50
60
Elementary
Kindergarten

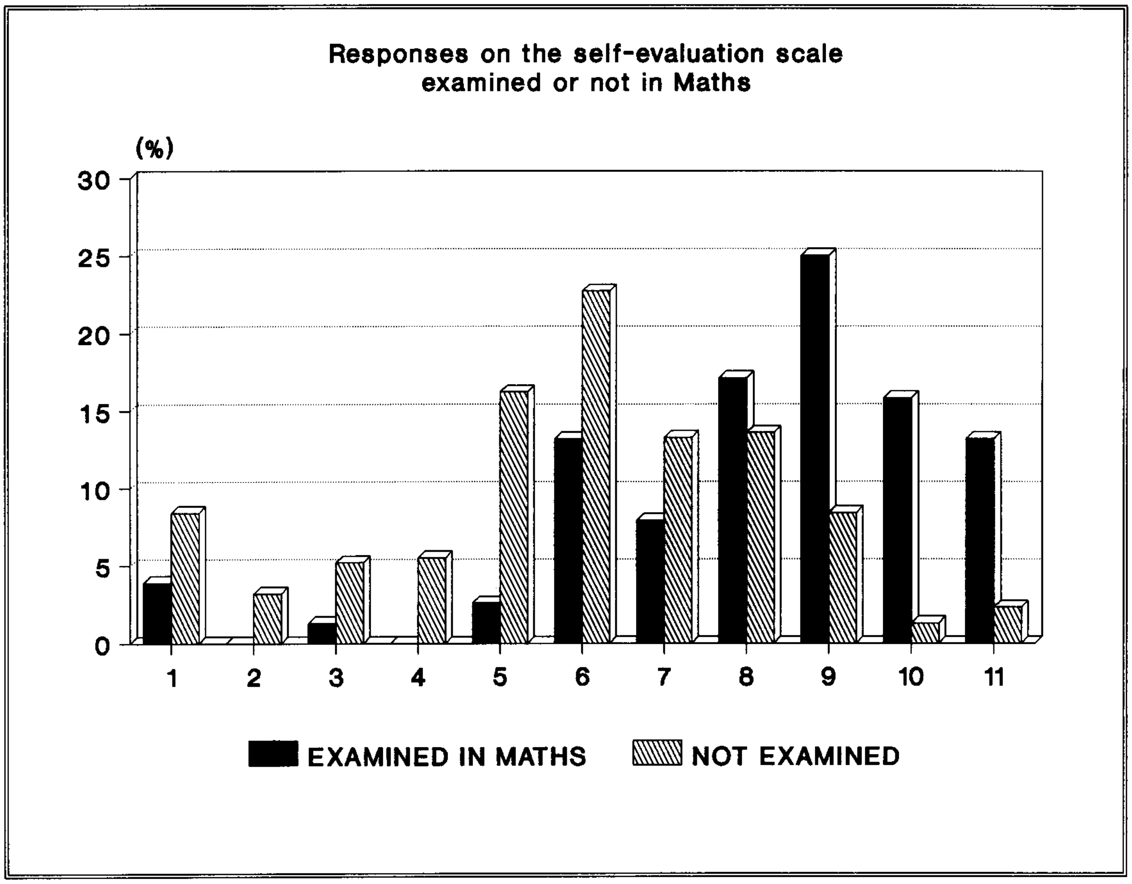

Responses on the Justification Scale
Reasons for disliking Mathematics
don't understand
cannot solve probl.
never done well
teachers not good
teachers not enthus.
unrelated to reality
lot of thinking
long work required
been afraid of them
exercises punishment
Frequency (%)
0
10
20
30
40
50
60
EXAMINED IN MATHS
NOT EXAMINED IN MATS


Responses on the Justification Scale
Reasons for disliking Mathematics
don't understand
cannot solve probl.
never done well
teachers not good
teachers not enthus.
unrelated to reality
lot of thinking
long work required
been afraid of them
exercises punishment
Frequency (%)
0
10
20
30
40
50
60
1st + 2nd YEARS
3rd + 4th YEARS

APPENDIX E



Responses on the self-evaluation scale
by major
(%)
25
20
15
10
5
0
%
1
2
3
4
5
6
7
8
9
10
11
ELEMENTARY
KINDERGARTEN


Responses on the self-evaluation scale
examined or not in Maths
(%)
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
11
EXAMINED IN MATHS
NOT EXAMINED


Responses on the self-evaluation scale
by level of studies
(%)
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
11
1ST + 2ND YEARS
3RD + 4TH YEARS