Paper Title: SPATIAL AND LOGICAL MISCONCEPTIONS IN LOGO SESSIONS: a comparison of normal and handicapped subjects' strategies
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Abstract: We created a new technique to observe and favor the cognitive development of young subjects: this technique is based on the use of concrete representations of formal systems (Lowenthal 1972; 1978). These representations consist of sets of objects provided with technical constraints. These constraints make certain actions possible and others impossible. Using simple devices as concrete representations of formal systems appears very useful for researchers interested in Cognitive Psychology: the technical constraints of the device provide hidden axioms and deduction rules. This creates a simple logical framework within which an objective interpretation of the subjects' productions becomes possible since all the ambiguities due to the use of verbal interactions can be eliminated. Lego bricks constitute a good example of a device that can be used as a concrete representation of a formal system; the bricks must be placed on a baseboard and a subject can only continue a path of bricks by placing more bricks next to the bricks already placed on the board, using flat or right angles: all other attempts yield to technical impossibilities. The axioms hidden in this device are those defining the absolute geometry. It must be noted that such devices can only be used to present a logical problem to the subject and to observe his or her solving strategy at the output level.

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Additional Note: The original graphics do not display properly in Common Ground for Windows 1.0. Neither the Misconceptions Seminar nor No Hands Software has been able to determine the reason for this problem, but we are working on it. To work around this problem in the meantime, we have rescanned the author's graphics and inserted these new graphics into the body of the article. The original versions of the graphics appear at the end of the article.
SPATIAL AND LOGICAL MISCONCEPTIONS IN LOGO SESSIONS: a comparison of normal and handicapped subjects' strategies.

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PRELIMINARIES.

We created a new technique to observe and favour the cognitive development of young subjects: this technique is based on the use of concrete representations of formal systems (Lowenthal 1972; 1978). These representations consist of sets of objects provided with technical constraints. These constraints make certain actions possible and others impossible. Using simple devices as concrete representations of formal systems appears very useful for researchers interested in Cognitive Psychology: the technical constraints of the device provide hidden axioms and deduction rules. This creates a simple logical framework within which an objective interpretation of the subjects' productions becomes possible since all the ambiguities due to the use of verbal interactions can be eliminated. Lego bricks constitute a good example of a device that can be used as a concrete representation of a formal system; the bricks must be placed on a baseboard and a subject can only continue a path of bricks by placing more bricks next to the bricks already placed on the board, using flat or right angles: all other attempts yield to technical impossibilities. The axioms hidden in this device are those defining the absolute geometry. It must be noted that such devices can only be used to present a logical problem to the subject and to observe his or her solving strategy at the output level.

Such concrete representations of formal systems were used with behaviourally perturbed children who rejected any form of schooling (Lowenthal 1972), and with many other children. The researches done with handicapped children lead to unexpected results. The children's reactions showed that this approach pleased them, that they accepted to attempt to solve logical problems presented in such a nearly non-verbal way, and moreover that they learned without difficulty what was taught via these concrete representations. Similar experiments with normal subjects gave similar results; they also showed that manipulations of a concrete representation of a formal system favours transfers towards more classical school domains: first graders manipulating the mechanical equivalent of the
hardware of a computer to discover the bases of finite automata theory learned significantly better (p < .05) to read (Lowenthal 1986b). We can thus claim that concrete representations of formal systems can be used to observe and favour the cognitive development of normal and handicapped subjects. These results raised a question: why does it work? The experimenter formulated an hypothesis to explain these progresses (Lowenthal 1986a): «Firstly, the use of a concrete representation of a formal system helps the subject to structure his perceptive field and to distinguish the relevant elements. In a second step, the subject will create small blocks of relevant elements. In a third step, a shift in the level of relevance will occur and the subject will start again the procedure described in the second step, but with a more global notion of relevance, until a general structure is reached».

In order to verify this hypothesis, further researches (based on the same technique) were organized. The fact that concrete representations of a formal system give data concerning only the subject productions at the output level appeared to be a serious limitation of the method. A partially computerized approach was used to solve this problem (Harmegnies & Lowenthal 1984): the problem was introduced by means of a computer screen and all the necessary information was made available in little boxes shown on the screen, but hidden behind «curtains»; the subject could use a mouse to raise the curtain of a given box and thus get hold of the necessary information. The computer was not only used as server, but also as observer as it kept track of the complete list of requests for information made by the subject, and of the exact time at which these requests had been made: the authors speak of an «ob-serving computer» (Lowenthal & Harmegnies 1986). The subject still had to manipulate concrete devices to solve the problem: the subject's productions were recorded by means of a video camera and a synchronization signal enabled the researchers to study in parallel the computer file containing the requests for information and the video documents containing the subject's productions.
INTRODUCTION.

The partially computerized approach gave more information than the first one since it enabled researchers to establish a link between the capture of a given information and its use in the production part of the problem solving activity; it also gave important information concerning the order in which information had been requested. This approach thus enabled the researchers to make certain inferences concerning the actual problem solving strategies by observing the list of repetitions of identical requests. This technique of observing computerized concrete representations of formal systems was then used in the context of strategy games (Lowenthal, Meunier & Willaye 1986). The software used for this experiment was based on the game of NIM. The population consisted of 18 children: 6 came from a very low socio-economic background, were mildly retarded and attending special education; the other ones were normal but 6 of them were considered by their teacher as the worst pupils in the classroom while the 6 remaining ones were considered as the best ones. Unexpectedly the retarded children obtained the best results for simple exercises, followed by the «bad» normal ones which were still better than the «good» normal students. None of the subjects succeeded for complex exercises. This experiment was repeated and the new results confirmed the old ones. This suggested that the use of our ob-serving computer technique might reveal the existence of unknown misconceptions concerning young subjects (9-year olds) in the framework of problem solving activities.

It was also decided to study the cognitive strategies used by normal and by handicapped subjects to create and/or to reproduce drawings, as well as the strategies they use for the acquisition and the creation of new concepts. Logo is a powerful computer language adapted as well for the creation of drawings as for the construction of new concepts: according to Papert (1980), a non directive approach of this language enables the child to learn how to «teach new words to the machine», and thus to develop his own conceptualization power. This is true for normal subjects; it is also true for handicapped subjects (Goldenberg, 1979). We thus chose to let our subject work in a Logo environment providing the type of information we needed: we created for the Logo language an adapted software (Lowenthal, Marcourt & Solimando 1993). In this paper we will first present this software, called the Logo observer, then describe our population and the methodology we used.
and finally present and discuss the results which have been observed.

THE SOFTWARE.

The Logo language as such cannot be used as computerized concrete representation of a formal system. Something is lacking: the classical use of Logo produces a result (a drawing, a program, a set of procedures, etc.) but gives no information concerning the mental processes involved in the child's production activity. Even when a correct result is produced, the richness of the language makes it impossible to infer from the result the problem solving strategy used by the subject. Moreover, in some situations, the great number of primitives available in Logo appears to be cumbersome and sometimes even seems to restrict the child's creativity. The richness of Logo, although very useful, makes its use as a concrete representation of a formal system impossible: the system is too flexible, requires subject-computer interactions in a pseudo-verbal language, and an understanding of classical Logo error messages which is difficult for youngsters with a communication problem and for young children who are still developing their verbal language.

We thus decided to create a version of Logo which could be used as a computerized concrete representation of a formal system: we decided to reduce the number of available primitives and to simplify the error messages. Our version is also an «ob-serving» Logo since it can be used as «server» by the subject while it is «observing» for us the subject's productions. Our version of Logo enables us to keep track of all the subject-computer interactions. This software is inspired by work done by Bois and discussed with him in 1977 (Bois et al., 1981) but goes beyond Bois' original ideas. The present version calls a program creating several variables. Some of them contain informations concerning the subject who is using Logo (name, etc.). There are also two major variables: the first one contains all the instructions given by the child to the computer via the keyboard (including the moment at which they were given), the second one contains all the successive definitions (procedures) invented by a child for a given concept until he discovered a satisfactory one. These variables are saved on a diskette at the end of the session. It thus becomes possible to observe the attempts made by children who eventually succeeded, within a purposely and logically restricted sub-language of the real Logo, but also the difficulties encountered by those who did not succeed. This enables the educator to provide each child with
appropriate help and new individualized problems taking into account the misconceptions revealed by the Logo observer. For the purpose of our research, we added to the set of information obtained by our Logo-observer, the video observations of the classroom: this enabled us to observe the whole child.

**POPULATION.**

We first conducted a preliminary experiment with handicapped adolescents, and later an experiment with 4 groups of normal 9-, 10- and 11-year olds. The results we present here concern both groups: comments will specify whether given data concern more the handicapped or the normal subjects.

The group of handicapped subjects consists of six 14- and 15-year old boys. All of them are mildly retarded and the IQ which are mentioned in their files are all below the normal: $65 \leq \text{IQ} \leq 78$. The test used (the WISC) provides the researcher with two IQ scores: a verbal and a performance one; for five subjects the verbal IQ is much smaller than the performance IQ. The situation is usually reversed with subjects «who learn well at school». All the subjects of our sample had important spatio-temporal problems associated with behavioural problems. They showed an important opposition to school activities and a serious lack of concentration. They followed a vocational training in plumbing, in a special education school.

The group of 79 «normal» subjects consists of four forms: one 4th grade (9-year olds), one 5th grade (10-year olds) and two 6th grades (11-year olds). The subjects are boys and girls who had no prior exposure to Logo (9- and 10-year olds) or a short exposure to Logo during the previous school year (11-year olds): they remembered some elements but did not know how to use them. All have an IQ > 100, and most have a verbal IQ higher than their performance IQ; they do not show any special opposition to school work and do not lack concentration.
METHOD.

There were 20 one-hour sessions, one session a week. The sessions took place in the school during normal school hours. The subjects worked in groups of two.

At the start the only primitives available to the subjects were the four functions **FORWARD (FD)**, **BACK (BK)**, **RIGHT (RT)** and **LEFT (LT)**. These functions act on a number: FD 100 provokes a forward move of 100 turtle-steps in the direction towards which the turtle is presently pointing, RT 45 provokes a rotation to the right of 45° using the present turtle direction as reference: these four basic functions govern the turtle moves. The primitive **CLEARSCREEN (CS)** was soon made available for our subjects: this primitive «cleans» the computer screen but keeps the memory unchanged.

We used the following 7-steps hierarchy of situations:

1) undirected play with the four primitives **FD**, **BK**, **RT** and **LT** during session 1;

2) copy of simple drawings (presented on grid paper for the handicapped subjects), such as the drawing shown in figure one, during sessions 2 to 5;

![Figure 1: Plane drawn using only right angles](image)

3) introduction of the concept of **procedure** during sessions 6 and 7 (handicapped, earlier for normal subjects);

4) a) presentation of situations requiring more than one simple procedure (such as HOUSE) during sessions 8 and 9 (earlier for normal subjects);

   b) guided construction of HOUSE40 and HOUSE70 (without variable)
during session 10 (for normal subjects: guided discovery);

5) introduction and use of the concept of **variable** during the three following sessions;

6) introduction of a procedure **ARC** producing a quarter of circle to the right
during a further session. ARC is defined as follows:  

\[
\text{REPETE 18 [ FD 3 RT 5] }
\]

a) undirected play and discovery of the potentialities of ARC during this session (similar to the undirected play with the four basic primitives during session 1);

b) guided discovery of complex procedures based on combinations of ARC, such as PETAL, BUTTERFLY, FLOWER, TULIP, etc.) during the following sessions (15 to 19 for the handicapped subjects);

7) introduction to the concept of recursion by presenting complex drawings: the TARGET shown in figure 4 (all subjects) and the BIG and the SMALL fractals shown in figure 2, during the last sessions (only for the handicapped and the «normal» 11-year olds).

**RESEARCH QUESTIONS.**

The Logo environment is based, among many other things, on manipulations, via a computer keyboard, of a small triangle called turtle. At the start, this «turtle» is in the center of the screen, pointing upwards. The turtle can be moved and these moves can be used to create drawings, but the turtle can also travel without leaving any track. The turtle moves refer to a relative coordinate system: in fact the present position and orientation of the turtle is the origine or «zero-point» from which the next movement will be computed, the arrival point and orientation defining then the new «zero-point». This type of relative coordinate system is similar to that of a human being walking in a city. Cordier (1975) described the type of problems met by some handicapped persons who ignore the shortcuts while walking and walk along the three sides of a square because they cannot conceive that walking along the fourth one will produce the same result more easily. Piaget (1972) claimed that a clear understanding of the inverse function, and even of his
INRC\(^1\) group is needed before one can assume that a subject has reached the level of formal operations. To keep making a detour by walking along three sides of a square would have been considered by Piaget as a very negative element in the child's cognitive development, he would have viewed this as a very serious spatio-temporal misconception.

Logo is a language devised to let children learn by teaching new words and new ideas to their computer. They can do so by defining procedures using the basic primitives existing within Logo. One of Papert's claims is that children using Logo will create procedures, use them as if they were primitives and include them in the definition of more complex procedures.

Drawings can be produced on the computer screen in very different ways: this is the case for the HOUSE shown in figure 3.

![Figure 3: The HOUSE](image)

The subject could (at least) choose one of the six following modes:

1) the **direct mode**: the subject only uses the basic Logo primitives;  

2) the **direct call of procedures**: the subject first defines procedures and then uses them in direct mode as if they were actual primitives. For the house he would define a square and a triangle of the same size (e.g. 40 turtle steps) and he would then produce the following list of instructions: [SQUARE FD 40 RT 30 TRIANGLE] which yields the required drawing;  

3) the **simple procedure**: the subject creates a concept HOUSE, but in the definition he uses **only** the basic Logo primitives he would have used in direct mode;  

\(^1\) Implication, Negation, Reciprocal and Contraposition
4) the **simple procedure with call** : the subject creates a concept HOUSE and in the definition he uses mainly basic Logo primitives to produce his drawing, but one or several procedures defined before are also used to produce lines on the screen. In the case of our house, this could lead to the following definition:

```
TO HOUSE
  SQUARE
  FD 40
  LT 90
  BK 40
  REPEAT 3 [FD 40 RT 120]
END
```

5) the **superior procedure** : the subject creates a concept HOUSE and in the definition he uses mostly previously defined procedures to produce his drawing: these procedures are linked by basic Logo instructions which are not used to produce elements of the drawing. In the case of a house similar to that described using the «direct call of procedures», this would require the definition of SQUARE and of TRIANGLE and would lead to a procedure defined, for instance, as follows:

```
TO HOUSE
  SQUARE
  FD 40
  LT 90
  BK 40
  TRIANGLE
END
```
6) the complex procedure: the subject creates a concept HOUSE and in the definition he uses only previously defined procedures. In the case of our house, this would require a definition of SQUARE and of TRIANGLE, but also a definition of the concept MOVE enabling the turtle to travel without leaving a trace from the end of the square to the place where it should begin to draw the triangle. This could give the following elegant definition:

```
TO HOUSE
  SQUARE
  MOVE
  TRIANGLE
END
```

It thus appeared relevant to examine whether the subjects had a correct (i.e. adult-like) understanding of the following elements:

a) space.
   1) Did the subjects understand that LT is the inverse functions of RT; that BK is the inverse of FD? Were they able to combine them in a fashion showing that they had understood the bases of the INRC group?
   2) How did they correct their mistakes? Did they systematically use the CS primitive or did they become familiar with primitives such as PENERASE (PE), PENUP (PU) and PENDOWN (PD)? Did they use other correction techniques concerning their turtle incorrect movements?
   3) How did they draw polygons? Did they realize that the turtle is turning around the shape it draws, and thus that, while drawing a polygon, the turtle is making a complete turn (360°): the relevant angles for the turtle are thus the exterior angles and not the interior ones. In the same context one can wonder whether the subject had a correct understanding of the concept of angle, of right angle, of equilateral triangle, etc.
   4) Did the subjects have a practical sense of proportions? Did they conceive that they should start some of their drawings on the side of the screen after moving the turtle without drawing? Were the handicapped subjects helped by the use of grid paper?
b) **logic:** i.e. the use, creation and combination of new concepts (procedures).

5) Did the subjects create new procedures? Did they do so frequently? Did they start in the procedural mode, or did they first test the situation using the direct mode, i.e. did they restrict themselves to use FD, BK, RT, LT, and some other pre-existing procedures?

6) Did the subjects realize that CS cleans the screen, replaces the turtle in its center in a starting position, but leaves the computer memory unchanged?

7) Which kind of cognitive strategy did they use to produce complex drawings, knowing that previously defined simple procedures were available?

![Figure 4: A target made of squares](image)

8) Were the subjects able to use adequately the concept of variable to create a target similar to that shown in figure 4? Were they able to understand that the five elements of the SMALL fractal of figure 2 are only reduced versions of the BIG fractal shown in the same figure?
RESULTS.
All the subjects were soon familiar with the Logo environment, but the handicapped and the normal subjects used this environment differently: we will thus present their results separately.

Handicapped subjects.
1) a) Until the last session, one finds sequences of instructions such as [RT 90 RT 90 RT 90] or [RT 90 RT 180] instead of LT 90. This occurred in direct mode only, but this mode was used during most of the time by these subjects.
b) Two subjects used during the first sessions [RT 180 FD 20] instead of BK 20. But these two occurrences were clearly exceptional and such a combination was never used in further sessions.
c) Eventually, all basic primitives (FD, BK, RT, LT, CS, PE, PU and PD) were correctly used.
2) a) During the two first sessions the subjects could only use CS to correct their mistakes. PE, PU and PD were introduced during the third session: since then the subjects try to correct their mistakes without erasing the complete drawing, CS is now only used when the correction to be done is too complex.
b) The subjects also discovered very early other correction techniques: the compensation (RT 85 followed by RT 5 to obtain the desired 90° rotation), the cancelation (RT 85 followed by LT 85 to come back to 0°), the list inversion ([FD 20 LT 15 FD 15 LT 90] followed by [PE RT 90 BK 15 RT 15 BK 20 PD] or even the PE transfer (PE was introduced for one basic instructions and the subjects discovered that it could be used for one complete procedure: [PE SQUARE PD]).
3) a) The notion of right angle was not mastered by these subjects. Several drawings requiring different kinds of right angles were presented (see figure 5) and the subjects were asked to reproduce them on their computer screen.
**Figure 5**: Example of drawing with many kinds of right angles

b) Triangles were drawned in direct mode, by trial and error. The following examples show that the subjects kept changing the angle magnitude **while** they were constructing their triangle:

- [RT 50 FD 50 RT 10 FD 50 RT 60 RT 20 FD 50]
- [RT 90 RT 45 FD 50 RT 90 RT 10 FD 50]

We told the subjects to stand up in the classroom and to trace, with their own body movements, a real triangle: they did it but did not transfer this type of information in terms of turtle moves and in terms of instructions to be given to the computer. The subjects did not discover by themselves how to create the concept TRIANGLE and we decided to give them the solution.

4) The subjects did not take into account the following information concerning the drawing shown in figure 5 and presented on grid paper: «**each square measures 5 turtle steps**». They worked by trial and error and obtained an acceptable result where proportions had been respected, but they did not first change the starting point. They never wrote down notes describing the instructions they had used to move their turtle on the screen (while leaving tracks or in PU mode). The subjects reproduced in their screen-drawings the **details** shown on the pictures we gave them: they did not appear to be concerned by the general appearance.

5) The subjects created new procedures, but they did not do it frequently, nor did they do it spontaneously: the concept of **procedure** did not seem to be of great use for them. They never started by writing a procedure: they always used the «direct mode» as test prior to the definition of a procedure. During their trials and errors in «direct mode» they only used the RT function to define rotations: they used three consecutive RT 90 instructions to obtain a LT 90. The **procedures** they defined after these tests contained RT 90 **and** LT 90 instructions: they were not a simple reproduction of the list successfully used in «direct mode».

6) Using CS after a procedure had been defined did not require, for them, a new definition of the procedure: they used CS as if it did not change the memory (which is correct).
Figure 6: FOUR_FLAGS

7) These subjects were asked to produce complex drawings on their screen knowing that several already defined procedures were available (e.g. SQUARE). They adopted unexpected strategies. They always started in direct mode. Knowing that SQUARE was available, they used in «direct mode» the following list of instructions to draw a FLAG:

\[
\text{[SQUARE BK 40]} \quad \text{and they used, again in «direct mode», the following list of instructions to create the drawing shown in figure 6:} \\
\text{[SQUARE BK 40 RT 90 FD 40 SQUARE BK 40 RT 90 FD 40 SQUARE BK 40 RT 90 FD 40 SQUARE]. They did not spontaneously create a procedure for FLAG nor a procedure FOURFLAGS. It should be noted that they did not use very often the REPETE function: the second list could have been reduced to } \\
\text{[REPETE 4[FD 40 SQUARE BK 40 RT 90]] or even better to} \\
\text{[REPETE 4 [FLAG]]. When asked to create a procedure for the house shown in figure 3, one of the subjects kept using the «simple procedure» approach but the five other ones started to use the «superior procedure» approach. One week later four of these five subjects (but not the two others) produced a definition of the «complex procedure» type, but the simpler procedures they used (inside their main definition) were only based on very basic Logo primitives. They defined SQUARE by typing the following list of instructions (one instruction per line):} \\
\text{[FD 40 RT 90 FD 40 RT 40 FD 40 RT 40 FD 40 RT 90] instead of using the automatic repetition produced by the REPETE function (which they new). As the four subjects who used this approach used identical definitions calling identical simpler procedures in the same very inelegant way (and as this «complex procedure» approach was never used again), we wonder whether these subjects had really discovered by themselves the definition they used or whether they had been «helped» by one of their regular teacher who}
\]

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wanted to impress us.

8) a) The subjects did not use adequately the concept of variable to produce the target shown in figure 4. Some restricted themselves to the «direct mode» approach (although SQUARE :SIDE had been defined, with :SIDE the variable used to define the length of the side), one of them used the «direct mode with call of procedures» approach; he produced a long list of instructions starting in the following way: [SQUARE 10 PU LT 90 FD 5 RT 90 BK 5 PD SQUARE 20 PU ...].

b) The subjects defined separately BIG and SMALL (see figure 2). In both cases they used the «simple procedure» approach. They did not use BIG to define SMALL. They tested BIG in direct mode before they defined it, but they started directly to define SMALL without any test.

Normal subjects.

We took only in consideration the results of the 43 9- and 10-year olds since the 11-year olds had already approached Logo earlier and since they were too close to the age foreseen by Piaget for a normal access to the stage of formal operations.

1) a) As for the handicapped subjects, one finds sequences of instructions such as [RT 90 RT 90 RT 90] or [RT 90 RT 180], but they are not frequent. Contrarily to what happened with the handicapped subjects, this attitude stopped for nearly all the subjects as soon as the concept procedure had been introduced.

b) We did not notice any occurrence of a [RT 180 FD 20] replacing a simple BK 20, but this might be due to a technical problem provoking the loss of some information during the first session.

c) All basic primitives were correctly used after the second session.

2) a) The subjects used CS during the first session. They asked for a «better» primitive as early as the second session: PE, PU and PD were then introduced. After that, the subjects restricted the use of CS to cases where the required correction was really too complex.

b) The compensation and cancelation technique could be observed as correction techniques. There is little evidence of the PE transfer. It must be noted that the normal subjects usually worked in one of the «procedural modes» described above: they mostly used procedures,
and «superior» or «complex» procedures calling simpler ones. As far as this working method is concerned, the complete definition is either correct or incorrect. In the latter case, the wrong procedure must completely be redefined (the software we use does not permit the use of the Logo editor). The PE transfer was thus not very relevant for the normal subjects; it was relevant for the handicapped subjects since they used a lot the «direct call of procedures» approach.

c) There is no evidence of list inversion for these subjects.

3) a) The concept of angle was mastered by all normal subjects, but the 9-year olds did not know exactly the magnitude of a right angle (this was not the case for the 10-year olds). Exercises such as «drawing the dog» (figure 5, given on plain - not guid - paper) were used for the 9-year olds. This simple remediation was sufficient to enable all the normal subjects to create squares.

b) The 9-year olds created triangles in direct mode, by trial and error. They tried and added different angle magnitudes ([RT 90 RT 20 RT 10]): most of them obtained the correct magnitude (120°), but a few of them changed magnitude inside a given drawing made in direct mode. They were told to stand up and turn around their teacher while tracing a triangle: this activity enabled them to realize that their body movements provide them with a good representation of the instructions they should give to the turtle. The 10-year olds started in one of the procedural modes, without prior test in direct mode. Most normal subjects, while creating a procedure for TRIANGLE, tried several angle magnitudes; in most cases they kept the angle unchanged within a given definition, they tried to execute it and they adapted the angle magnitude if the definition was incorrect. Eventually all discovered how to create the concept TRIANGLE.

4) When the subjects reproduced the drawing shown in figure 5 and given on ordinary paper, they worked by trial and error. Some 10-year olds (but no 9-year old) decided to note on their piece of paper the size of the turtle moves they made, but they did not seem to use this type of information: their drawings did not respect the imposed proportions. They did not really try to respect the details of our drawing, but they did the best they could to produce a drawing which conveyed the same idea.
5) All subjects created numerous new procedures: as soon as this possibility was introduced they abandoned the direct mode. The procedures created by subjects using the «direct mode» as test contained exactly the list of instructions which had been tested, including sequences of three consecutive RT 90 replacing a LT 90.

6) Using CS after the definition of a procedure seemed to require a new definition of the procedure, for the 10-year olds but not for the 9-year olds.

7) When asked to produce complex drawings, the 9-year olds started by a test in direct mode (this was not the case for the 10-year olds). In a second step, all normal subjects used the «direct call of procedures» approach; they used then the «simple procedure» approach or even the «superior procedure» approach: in this case their definitions of simpler procedures used an economical list of basic instructions (such as [REPETE 4[FD 40 RT 90]] to define SQUARE).

8) a) Most 9-year olds started to draw their target in «direct mode» and then they used (as the 10-year olds immediately did) a «direct call of procedures». The 9-year olds called the procedure SQUARE :SIDE and most 10-year olds called the procedure POLY :SIDE :NUMBER (defining any polygon) to create the visible part of their drawing; they also used basic Logo instructions but they restricted the use of such instructions to describe the necessary «invisible» turtle moves. Some 9-year olds created a procedure MOVE describing these trackless turtle moves; they then created their drawing as follows: [SQUARE 20 MOVE SQUARE 40 MOVE SQUARE 60 MOVE SQUARE 80]. Eventually all normal subjects created a procedure for their TARGET: they used either a «simple procedure with calls» or a «superior procedure» approach. The simpler procedure which was called always contained at least one variable which was always adequately used.

b) As the school year ended, and as the teachers and the subjects appeared more concerned by exams, and thus too nervous to make serious discoveries in the Logo environment, we delayed the exercise concerning the fractals BIG and SMALL (and similar ones) until the next year.

DISCUSSION
1) The 14- and 15-year old handicapped subjects did not seem to realize that LT is the inverse function of RT, but they seemed to have no misconceptions about the relation between FD and BK. They seem to have a very incomplete understanding of the INRC group. This does not seem to be case of the normal subjects, although these subjects have not yet reached the age mentioned by Piaget for the stage of formal operations.

2) All subjects had a good understanding of the proposed correction techniques. The handicapped subjects invented unexpected techniques adapted to their needs; the normal subjects did not seem to need special techniques: they used the sophisticated construction techniques provided by Logo.

3) The handicapped subjects did not understand the concept of angle. Drawing polygons was thus a real problem for them. On the contrary, all normal subjects understood the concept of angle and the fact that the turtle must make a complete turn in order to draw a polygon; the only problem was due to a lack of knowledge in the youngest normal subjects. All normal subjects discovered that their own body movements gave a precise representation of the instructions required by a two dimensional turtle moving on a computer screen, and conversely: one could see these subjects bending their head one way or the other in order to discover what was the instruction they had to give; this type of transfer never appeared in our handicapped subjects and this is an important misconception of the world: the handicapped subjects who learn plumbing do not realize that their body should be their main orientation reference.

4) The handicapped subjects were helped by the fact that the drawings we presented to them (and asked them to reproduce on the screen) were printed on grid paper: one can say that they had a good sense of proportions, even if the techniques they used to create their drawings were very inelegant. The situation was reversed for the normal subjects: we presented them with the same drawings, but printed on ordinary paper; the techniques they used to create their drawings on the screen were very elegant but they seemed to lack the sense of proportions. We do not know whether they would have performed better had the drawings be given on grid paper. The handicapped
subjects limited themselves to a precise reproduction of the details, contrarily to the normal subjects who limited themselves to an unprecise representation of the general idea conveyed by the drawing. We feel that, in both cases one should speak of **misconceptions**, but these misconceptions are of a very different nature. In the handicapped subjects, the misconceptions are at a meta-level: they do not realize that the most important thing is to produce an image which will transmit to others the concept we had in mind when we created the drawing; they might become a good helper for a specialist in plumbing, but they will not be able to organize by themselves a set of tubes in order to have «everything that is needed» for a new house. In the normal subjects, the misconceptions are due to their young age at the object-level: they do not realize that the general idea is **not** enough, that many technical details must also be specified; but this seems to be and teachable provided one takes into account what is known about their incorrect understanding of the situation.

5) The handicapped subjects did not have a good understanding of the concept of procedure. They did not create many new ones, and they always tried first in direct mode. All normal subjects had a good understanding of the concept «procedure», but the 9-year olds lacked some of the knowledge the 10-year olds had: as a consequence, the first **productions** of the normal 9-year olds can be compared to those of the handicapped 14- and 15-year olds; nevertheless there is a **qualitative** difference in favour of the 9-year olds, difference which becomes bigger as soon as they acquire the necessary knowledge: then they perform as well as the 10-year olds. Here one must use the term **misconception** for the handicapped subjects: the handicapped subjects think in terms of the «direct mode» approach even when they create a procedure, while the normal subjects think in one of the «procedural modes», even when they try first in «direct mode».

6) As mentioned above, all subjects used CS adequately and the misunderstanding observed in the 10-year olds (CS also destroys the computer memory) was only a lack of knowledge.

7) The strategies used by the handicapped subjects to create complex drawings do not seem to be as rich as those used by the normal subjects, nor do they reach the same level of complexity (except once
but ...). Is this a misconception or a lack of knowledge? The fact that handicapped subjects produced once a complex procedure for their house is not satisfactory. The fact that they tried all their procedures in «direct mode», before thinking about them in procedural terms is a more negative element for the handicapped subjects; this was not the case for the normal 10-year olds. Even when the 9-year olds «tried» their future procedures in direct mode, they were thinking about them in procedural terms. Here again, the productions of the handicapped subjects and those of the normal 9-year olds look very much alike, but the process used to reach them is fundamentally different. What is probably a misconception in the case of the handicapped adolescents is probably due to a lack of training in the case of the normal 9-year olds.

The fact that the handicapped subjects did not use variables in an adequate way shows that these subjects have a very serious misconception concerning a notion which is regularly used during their mathematics courses. This is clearly not the case of the normal subjects. Similarly, the non identification of the general shape of BIG and SMALL (and other similar shapes) must be considered as a misconception: the handicapped subjects appear unable to generalize a notion. We do not have data concerning the fractals for the normal subjects, but their attitude when confronted to the creation of the target lets suspect that there would have been no misconception in their case.
CONCLUSION.

On the one hand, there does not seem to be striking differences between the productions of handicapped 14- and 15-year olds using Logo and those of normal 9- and 10-year olds, on the other hand a comparison of the processes the subjects use show great differences: major misconceptions in the handicapped subjects (which confirm Cordier's observations), versus a possible lack of knowledge in the normal subjects. Such an observation of the processes used is easy and reliable thanks to our Logo observer.

Piaget described the evolution of the child in terms of stages, the highest one being the stage of formal operations. This author observed that this stage is usually (but not always) reached by 12-year old normal subjects; he also described a group, which he called the INRC group, and claimed that subjects who do not have a complete mastery of this group have not reached the stage of formal operations. The INRC group is generated by the combinations of four operators (Implication, Negation, Reciprocal and Contraposition) acting upon the formula \(( p \Rightarrow q )\); this group is equivalent to the group of transformations of a square generated by 90° rotations and axial symmetries). Our handicapped adolescents do have not mastered the bases of this group, but all our normal 9- and 10-year old subjects have good and solid notions of the bases of this group (although they are less than 12 years of age). The good results obtained by the normal subjects suggest a first research question: «do normal subjects manipulating concrete representations of a formal system, computerized or not, progress faster in terms of Piagetian stages than comparable normal subjects using other devices?». Any answer to this question must be interpreted in Brunerian terms (Bruner, 1966) rather than in Piagetian ones (Piaget and Inhelder, 1959).

A positive answer to our first research question will imply a second research question: «can manipulations of our Logo observer, or of any other concrete representation of a formal system, be used as a remediation for handicapped subjects?»

Positive answers to both questions will provide us with a confirmation of the hypothesis we formulated at the very beginning of this paper.
BIBLIOGRAPHY.


Original Graphics

Please note that these graphics do not display properly in Common Ground 1.0 for Windows. No Hands Software is working on fixing this problem.

Figure 1: Plane drawn using only right angles

Figure 2: the BIG (left) and SMALL (right) fractals
Figure 3: The HOUSE

Figure 4: A target made of squares

Figure 5: Example of drawing with many kinds of right angles
Figure 6: FOUR_FLAGS