Third Misconceptions Seminar Proceedings (1993)

Paper Title: **Misconception in Learning Differentiation**
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Abstract: In learning new calculus concepts, students are usually expected to have a considerable knowledge and understanding of certain basic mathematical concepts. These basic concepts are frequently used in explanations and definitions of new ideas and concepts. For example, in the study of differentiation, the concepts of gradient and tangent line are the underlying concepts for differentiation.

Keywords: concept Formation,Theories,Teacher Education,Concept Formation,Mathematical Concepts,Misconception,Theory practice relationship,Teaching for concepted change,
General School Subject: Mathematics
Specific School Subject: Calculus
Students: College Sophmores

Macintosh File Name: Ismail - Differentiation
Release Date: 12-15-1993 C, 11-5-1994 I

Publisher: Misconceptions Trust
Publisher Location: Ithaca, NY
Volume Name: The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics
Publication Year: 1993
Conference Date: August 1-4, 1993
Contact Information (correct as of 12-23-2010):
Web: www.mlrg.org
Email: info@mlrg.org

A Correct Reference Format: Author, Paper Title in The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Misconceptions Trust: Ithaca, NY (1993).

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Misconception in Learning Differentiation

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1. INTRODUCTION

In learning new calculus concepts, students are usually expected to have a considerable knowledge and understanding of certain basic mathematical concepts. These basic concepts are frequently used in explanations and definitions of new ideas and concepts. For example, in the study of differentiation, the concepts of gradient and tangent line are the underlying concepts for differentiation.

Without sufficient understanding and knowledge of the basic concepts, new concepts may be falsely interpreted or difficult to accept. It is therefore important that students have the correct image or perception on the concepts which will act as tools or foundations in building new more complex concepts.

What students see in a particular concept in term of mental pictures and associated properties and processes is termed as 'concept image' by Tall (1986). Tall termed 'concept definition' for the concept image that is expresed in a form of words used to specify that concept. Whatever lies in an individual's mind may not be expressed completely when put into words. This means that although concept definition describes an individual's concept image, the concept definition may not satisfactorily represent the real concept image. However, concept definition could be important in diagnosing a student's conceptual understanding and knowledge.

2. STUDENT'S BACKGROUND

A study aiming at diagnosing students concept image and concept definition on certain basic concepts of differentiation took place at Universiti

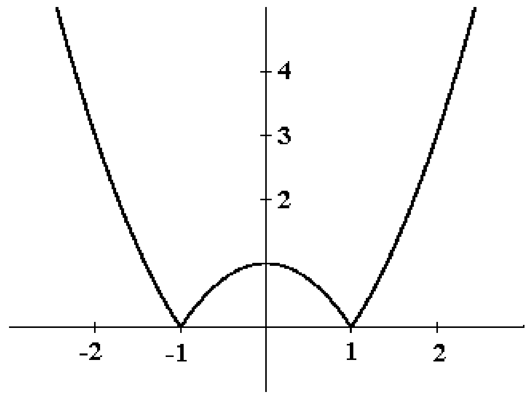
Teknologi Malaysia. 27 second year students enrolling in the science and mathematics teacher program have learnt differentiation at three different stages. Their first encounter with differentiation was during their final year in secondary school. The second stage was during their first year at the university and finally during their second year at the university. At the secondary school, differentiations were introduced by looking at limit through examples, definition of the first principal, rules and theorems in differentiation. applications of differentiations in problems involving tangent, rate of change and extremum points. The learning of these topics are repeated in the first year university level so as to ensure their understanding and knowledge in calculus before they continue their studies in physics, chemistry or other higher level mathematics. These students again studied differentiation during their second year at the university. This time they look at differention by extending the concepts, rules and procedures to inverse trigonometric functions, hyperbolic functions and inverse hyperbolic functions. They were also introduced to Leibnitz theorem. Calculus are viewed to be important for these students to support in their studies and in their proffesional career as educators.

This study was conducted while they were enrolling in the third stage of learning differentiation.

3. THE CONCEPTUAL QUESTIONS

Students participated in the study were given a test that was designed to determine their concept definition and concept image on tangent line, gradient and derivative. It is believed that these students have never before encountered the questions in this test. All of the questions except questions 4 were cited from Tall (1986) and are stated as follows:

Question 1
What is meant by a tangent to a graph?
Question 2
What is meant by the derivative of the functions?
Question 3
Given is a graph of $y = |x^2 - 1|$



(i) Which of the following is true?

a) The graph has no gradient at x = 1
b) The graph has one gradient at x = 1
c) It has two gradients at x = 1
d) It has more than two gradients at x = 1
e) Other comment(specify)

(ii) Which of the following is true?

a) The graph has no derivative at x = 1
b) The graph has one derivative at x = 1
c) The graph has two derivatives at x = 1
d) The graph has more than two derivatives at x=1
e) Other comment (specity)

(iii) Which of the folowing is true?

a) The graph has no tangent at x = 1
b) The graph has one tangent at x = 1
c) The graph has two tangents at x = 1
d) The graph has more than two tangents at x = 1
e) Other comment (specity)

Question 4

State 2 cases where the derivative of a point does not exist.

4. ANALYSIS OF RESPONSES

Responses from the test papers of the 27 students were graded and analyzed. It is found that some students did not response to all the questions. The responses from the open ended questions were divided into different categories according to the statements written by the students. However vague responses were not included in the categories. Type of responses and its frequency are listed below.

Responses from question 1

Type of responses		frequency
1a.	A straight line drawn perpendicular to the graph.	12
1b.	A straight line drawn touching or passing through one point only.	5
1c.	A straight line drawn by joining two points that are very closed together.	2
1d.	A line drawn on a graph and the slope of this line determine the gradient of the function.	2
1e.	A straight line that is horizontal to a given point	1
1f.	A straight line that is drawn on a graph that is not straight.	1

Responses from question 2

Type of responses		frequency
2a.	The gradient of the graph.	8
2b.	A new function derived by differentiation.	5
2c.	The gradient of the points on the function.	4
2d.	Rate of change of y with respect to x.	1
2e.	The gradient of the tangent on the graph.	1
2f.	$\frac{d}{dx}f(x)$	2
2g.	A method to determine the gradient of a function.	1

Responses to question 3(i)

Type of responses	frequency
a	10
b	8
c	6
d	1
e	1

Responses to question 3 (ii)

Type of responses	frequency
a	10
b	11
c	3
d	0
e	1

Responses to question 3 (iii)

Type of responses	frequency
a	12
b	10
c	1
d	2
e	0

Responses to question 4

Type of responses		frequency
4a.	Exist asymptote or infinite at x = a	7
4b.	Not continuous at the point	6
4c.	Straight line or vertical line	3
4d.	A constant function	2
4e.	Undefined at that point	1
4f.	The gradient at that point is zero	1
4g.	A sharp edge on the graph	1
4h.	Limit does not exist	1
4i.	At a point (0,0)	1

5. DISCUSSIONS

Responses from all the questions showed that concept definition and concept image related to differentiation do exist in the minds of the majority of the students participated in the study. However, since the questions posed are considered basic to these second year university students, it is quite a surprise to find that there exist some students who have no idea at all to express in words the meaning of tangent and derivative. There is also a considerable number of students who have poor concept image and concept definition on differentiation.

For question 1, only six different categories of responses could be understood. The first your although were not expressed satisfactorily were accepted as correct. Unexpectedly, students wrote two wrong ideas about tangent as stated in 1e and 1f. A satisfactory answer for question one should be "a tangent at a point is a straight line drawn by joining two very closed points". This response is represented by 1a and only two students expressed their concept definition this way. The most popular response is 1a. The answer is considered partly correct because the idea of perpendicularity was not mentioned to be related to circle. The proper way to express the idea is "a line that is perpendicular to the circle that touches the point on the graph".

All of the responses categorized for question 2 are accepted although they are not expressed satisfactorily. 2c which is considered as the best response represent concept definition from four students only. 2g is considered as the least preference to be accepted as correct. Quite a few students gave responses 2b and 2f. This type of responses suggest their preference for viewing differentiation algebraicly rather than geometricaly.

Question 3 is related to question 1 and 2 because it requires understanding on the definition of tangent and derivative. Responses showed that many are confused with the number of tangents, derivative and gradients. The concept definitions of tangent and derivative from some students are not consistent with their concept image. This can be seen as their responses from question 3 do not support their ideas from responses of question 1 and 2.

Finally, question 3 gave the highest number of different responses for open ended questions and also gave the highest number of wrong concepts. 4c, 4d, 4f and 4i are all wrong answers. Unexpectedly as many as three students have the idea that derivative do not exist for a straight line or vertical line. They might have forgotten that vertical line in the form of x = a is not a function and derivative does not apply here. Another response which put the idea that derivative does not exist at (0, 0) might be due to the students mind is focused to the case of $f(x) = |x|$

Obviously, this study showed that students who are already familiar with differentiation still could not grasp completey the basic concepts. Some may have the proper concept image but could not express it with words in a satisfactory way . Some may be able to express in an accepted way but are still confused when applying the concepts to solving problem. Therefore, in order to improve learning, it is suggested that educators ensure that their students have acquired understanding and knowledge of basic concepts before introducing higher level concepts.

6. REFERENCES

Tall D.O., Building and testing a cognitive approach to the calculus using interactive computer graphics, Ph.D Thesis, (1986) Warwick University.