Third Misconceptions Seminar Proceedings (1993)

Paper Title: Promoting self-control of mathematics learning for pre-service primary teachers

Author: Hill, Lola

Abstract: The qualitative study on which this paper is based investigated the impact of a mathematics method program on pre-service primary teachers' relational understanding of mathematics, independence as mathematics learners, beliefs about their capabilities as mathematics learners, and mathematical skills. The program, based in primary schools, enables students to practise in an integrated and concrete manner what they are learning in theory and to reflect on their experiences. The program is influenced by Skemp's (1979) theory of intelligent learning, Perry's (1981) scheme of intellectual and ethical development, Knefelkamp's (1981) Developmental Instruction Model (DIM) based on Perry's scheme, and Posner, Strike, Hewson and Gertzog's (1982) conditions for conceptual change. The findings suggest that pedagogical strategies consistent with the DIM—particularly experiential learning and personalism—tend to foster the establishment of the conditions for conceptual change—dissatisfaction with existing ideas, and the plausibility, intelligibility, and fruitfulness of the new ideas. The year-long cycle of theory, practice, and reflection in which students engage appears to provide a powerful incentive for students to commit themselves to relational learning and teaching in mathematics via a democratic model. It appears that the personal, egalitarian nature of the program tends to allay students' fears about learning mathematics and enables them to feel more comfortable about asking questions and experimenting with ideas.

Keywords:
General School Subject:
Specific School Subject:
Students:

Macintosh File Name: Hill - Mathematics
Release Date: 10-16-93 A, 11-5-1994 I

Publisher: Misconceptions Trust
Publisher Location: Ithaca, NY
Volume Name: The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics
Publication Year: 1993
Conference Date: August 1-4, 1993
Contact Information (correct as of 12-23-2010):

Web: www.mlrg.org
Email: info@mlrg.org

A Correct Reference Format: Author, Paper Title in The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Misconceptions Trust: Ithaca, NY (1993).

----- -----

Promoting self-control of mathematics learning for pre-service primary teachers

Lola Hill, School of Mathematics, Science and Environmental Education
Deakin University Burwood Campus Melbourne

Paper presented at the Third International Seminar on Misconceptions and Educational
Strategies in Science and Mathematics
1st-4th August 1993 Cornell University, New York

ABSTRACT

The qualitative study on which this paper is based investigated the impact of a mathematics method program on pre-service primary teachers' relational understanding of mathematics, independence as mathematics learners, beliefs about their capabilities as mathematics learners, and mathematical skills. The program, based in primary schools, enables students to practiseanner what they are learning in theory and to reflect on their experiences. The program is influenced by Skemp's (1979) theory of intelligent learning, Perry's (1981) scheme of intellectual and ethical development, Knefelkamp's (1981) Developmental Instruction Model (DIM) based on Perry's scheme, and Posner, Strike, Hewson and Gertzog's (1982) conditions for conceptual change. The findings suggest that pedagogical strategies consistent with the DIM—particularly experiential learning and personalism—tend to foster the establishment of the conditions for conceptual change—dissatisfaction with existing ideas, and the plausibility, intelligibility, and fruitfulness of the new ideas. The year-long cycle of theory, practice, and reflection in which students engage appears to provide a powerful incentive for students to commit themselves to relational learning and teaching in mathematics via a democratic model. It appears that the personal, egalitarian nature of the program tends to allay students' fears about learning mathematics and enables them to feel more comfortable about asking questions and experimenting with ideas.

THE PROBLEM

This paper addresses a major problem in teacher education in mathematics. Evidence suggests that teacher education in mathematics is a relatively weak intervention (Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992; Skemp, 1979; Speedy, Fensham & Annise, 1989; Ball, 1988; Evans, 1987; Dettrick, 1981) in at least three ways. First, pre-service teacher education in mathematics appears to have only a limited effect on student teachers' willingness to learn and teach mathematics for relational understanding. Relational understanding, in Skemp's (1989) terms, is "knowing both what to do and why" (p. 2), and may be distinguished from instrumental understanding—the application of rules without reason. Second, it appears to have little effect on student teachers' willingness to teach democratically and to encourage children to take control of their mathematics learning (Skemp, 1979; Speedy et al., 1989). Third, teacher education in mathematics has only limited success in encouraging students to adopt more positive attitudes towards mathematics, in particular a more independent attitude towards learning and teaching mathematics (Speedy et al., 1989). The present paper focusses on the first and second issues and is based on findings from a study which addresses the wider problem (Hill, 1993). These issues are important not least because only if teachers have a sound relational understanding of mathematics, will they be able to teach for relational understanding and be confident enough to encourage children to take control of their mathematics learning. Also, substantial enhancement of students' relational understanding of mathematics is likely to result in improvement in their concepts of themselves as mathematics learners.

THEORETICAL FRAMEWORK

Perry's (1981, 1988) scheme of intellectual and ethical development outlines the developmental process by which relational and democratic approaches to learning and teaching—indeed to life generally—might be achieved. Perry's scheme portrays a continuum of development from dualism, characterised by a view of phenomena in black or white terms, to commitment to a relativistic understanding of human behaviour as influenced by context and interpretation. A view informed by committed relativism accepts that agency is within each individual, that we are responsible for our own decisions and behaviour, and that appropriate behaviour in complex situations cannot be determined by recourse to rules. A dependent, instrumental approach to learning is consistent with dualism while an independent, relational approach corresponds with a commitment to relativism. Many students react to the discovery of relativistic thinking, however, with profound anxiety (Perry, 1981).

What is the cause of this anxiety? Commitment to relativism entails accepting a high degree of personal responsibility in the knowledge that uncertainty and discomfort are permanent aspects of life (Perry, 1981). The dualistic antithesis is that if the correct rules are learned and applied the reward is certainty and complacency about the correctness of past, present, and future decisions. We need not accept responsibility to analyse and weigh evidence, nor to make our own judgements, nor to question authority, including our own authority once it has been earned. Thus, according to a dualistic view, students have fulfilled their responsibility as learners to the extent that they have memorised the given rules and completed the required exercises, and teachers need only hand down the correct rules and methods and identify students' right and wrong answers. Students operating at a dualistic level tend to demand recipes and rules to memorise and may become distressed or hostile when offered food for thought instead (Davis 1985; Tanner, 1989; White, 1988). Teaching aimed at a higher level may be regularly ignored or, worse, interpreted in the opposite way to that intended.

In modern Western cultures mathematics is taught almost without exception dualistically and instrumentally (Skemp, 1975; Bishop, 1988; Pateman, 1989; Kline, 1977; Sweller & Low, 1992; Bennett, Desforges, Cockburn & Wilkinson, 1984; Schoenfeld, 1988). Evidence suggests that these traditional approaches are failing to help students establish sound conceptual understanding (Schoenfeld, 1987, p. 24; Davis, 1984). The majority of students' misconceptions about mathematical processes, in my opinion, result from a lack of sound conceptual groundwork, and as such, are an inevitable consequence of students' experiences of instrumentalist school mathematics teaching. In Skemp's (1975) view the damage inflicted by instrumentalist teaching and learning approaches is considerable: "To the extent that what is being communicated is not intelligible, the receiver is trying to accommodate his schemata to assimilate meaninglessness. To do this would be equivalent to destruction of these schemata: the mental equivalent of bodily injury" (pp. 116-17).

How does the Perry scheme relate to the learning and teaching of mathematics? A relativistic approach to mathematics learning entails understanding that there are various ways of solving mathematical problems, that often there is a range of sensible solutions, that the application of various tools is legitimate—tools including logic, commonsense, intuition, visualisation, diagrams, models, role-play—and that everyone is capable of applying these tools. A relativistic understanding includes the knowledge that many mathematical problems may be interpreted in various ways, and that some methods or solutions may be considered more satisfactory than others depending on the chosen criteria.

The Developmental Instruction Model (DIM) (Knefelkamp, 1981), based on Perry's scheme of intellectual and ethical development, is designed to accelerate students' intellectual development via a blend of challenge and support; a blend which is modified continually during the program depending on the rate of students' intellectual growth (Shearn & Davidson, 1989). Four variables involving challenge and/or support are matched to learning styles characteristic of students operating at particular positions on the Perry scale. The variables are structure, diversity, experiential learning, and personalism. The structure variable represents the degree of direction provided for students, diversity represents the number and complexity of perspectives or alternatives offered, experiential learning represents the degree of involvement and concreteness of activities, and personalism indicates the degree to which the class offers a forum for cooperation, risk-taking, and critical and evaluative discussion.

The essence of providing support for intellectual growth is to clarify the rules of the game so that students are able to understand what is expected of them and are able to monitor their own progress towards success. The essence of provoking intellectual development through challenge is to encourage risk-taking, experimentation, and the tackling of new and complex ideas and tasks with enough support to eliminate students' fear that their mistakes may result in irrevocable failure. Without challenge, little of lasting value is learnt; without support, hope for success is lost. The difficulty of the teacher's task lies chiefly in determining and continually adjusting the blend to suit the changing needs of individual students.

To the task of persuading students to adopt relational approaches to learning and teaching mathematics, evidence suggests that a conceptual change approach is required (Handal & Lauvas, 1987; Zarorik, 1990; Sirotnik, 1983). According to Posner, Strike, Hewson and Gertzog (1982), several conditions must be fulfilled before learners will commit time and energy to conceptual change. What is required before such a commitment is made is that students be aware of and dissatisfied with their current conceptions and see that the new idea is intelligible and plausible and that taking it on is a fruitful proposition. In other words, before accepting that change is necessary, learners need to have lost faith in the capacity of their current conceptions to solve their current problems, be able to understand the new ideas, believe that the new ideas will solve the problem, and feel that it is worthwhile to put time and effort into learning the new ideas. The theory sounds simple: it is, however, more easily described than practised.

Applied to mathematics education the theory initially raises more questions than it answers. As long as students believe they are incapable of learning mathematics, for example—as is the case with many pre-service primary teachers—the question of taking responsibility for doing so is irrelevant. Thus the process of taking responsibility and achieving autonomy as a mathematics learner will depend on whether the student begins to believe that he or she is capable of learning mathematics. In the present program, therefore, the condition of plausibility of change takes on a different and greater significance than that probably intended by Posner et al.

Initial surveys of students entering the primary teaching course at Deakin University's Toorak Campus during the period 1989-1991 demonstrated that approximately two-thirds to three-quarters of each entry group harboured negative attitudes regarding their abilities as learners of mathematics and towards mathematics generally (Hill, 1993). Less than 10% of each entry group demonstrated mastery of mathematical concepts and skills at year-6/7 level (Hill, 1993). These findings are not incompatible with those from similar studies (Howard, 1989; Sullivan, 1987; Anderson, Crawford & Sinclair, 1989; Watson, 1987; Freeman, 1986; Ziukelis, 1990; Speedy et al., 1989). The present mathematics program, therefore, presents an appropriate forum in which to apply a conceptual change approach to mathematics learning. Conceptual change, in this instance, refers to students' adoption of a more independent, relativistic, and positive approach to learning and teaching mathematics, and a commitment to, and competence in, learning and teaching mathematics for relational understanding.

It is my opinion that the application of the Developmental Instruction Model is likely to enhance the establishment of the conditions for conceptual change. The study discussed in this paper was designed in part to monitor the degree to which this expectation was fulfilled. The mathematics method program on which the study was based is also consistent with recent theory and research regarding effective teacher education. The research of Batten, Griffin and Ainley (1991), Fullerton (1992), Borko, et al. (1992), and Tisher and Klinzing (1992) regarding the development of pre-service teachers' pedagogical understanding and skills emphasises the significance of a continuing cycle of theory, practice, and reflection. Improvement of teaching skills, according to Tisher and Klinzing, is related to teachers' opportunities to progress through a series of cycles in which they relate their theoretical understanding of teaching and learning processes to specific, concrete, and practical examples, and reflect on the outcomes.

APPLICATION OF THE DEVELOPMENTAL INSTRUCTION MODEL

The Deakin Burwood first-year mathematics method program is integrated with the language method program and is based in primary schools. This mathematics and language (MALL) program enables approximately 150 students to spend one day per week throughout the academic year in a primary school where they attend mathematics method and language method classes run by their university lecturers, and also teach small groups of children. The program, therefore, has forged a strong connection between the university and schools. The present study focussed on the mathematics component of the program. Following is a description of the four variables of challenge and support employed in the DIM and how they are structured in the program. For students with a dualistic outlook towards mathematics, high degrees of structure, experiential learning, and personalism provide support, while a high degree of diversity provides challenge.

The Structure variable represents the amount of direction given to students. In the present program students are provided with a comprehensive and user-friendly program manual, program goals and their relevance to the students are discussed and students are encouraged to set their own goals. Lecturers constantly demonstrate the relevance of the concepts and activities to the task of teaching mathematics to primary children. Each week students implement their ideas in planned micro-teaching sessions with small groups of children and subsequently participate in discussion and reflection regarding their experiences. Because, in each semester, the students visit the same classroom weekly, they become familiar with the children, the classroom teacher, and the general organisation of the class and school. The above factors provide a high degree of structure and, thereby support, for students.

Experiential learning refers to the degree of direct personal involvement in and concreteness of the program activities. Because the present program is school-based, theory and practice are closely integrated and there is a high degree of experiential learning which provides substantial support for students. Importantly, students have considerable control over what and how they teach; the content is determined by students in collaboration with their lecturers and classroom teachers. Because the content is closely related to what they learn in their university classes, students continually practise, reflect on, and modify the theory they are learning. In such circumstances it is easier for students to understand the relevance of what we are teaching them in their method classes. Students are required to use materials and commonsense approaches in order to build their relational understanding of mathematical processes, and to practise teaching with those materials and methods.

Personalism refers to the degree to which the class supports its members in asking questions, taking risks, voicing opinions, experimenting with ideas and processes, and showing enthusiasm for learning. An emotionally safe learning environment is established and maintained in which no disparagement is accepted. We expect students to demonstrate publicly their approaches to problems even when they are not sure about the outcomes of their attempts. A high level of energy and involvement from students and commitment to promoting their own and others' learning is expected.

Diversity represents the number and complexity of perspectives or alternatives which are offered or encouraged in the program. The present program presents a high degree of diversity in regard to ways of understanding mathematics and of helping children learn mathematics. We encourage students to develop their own methods for teaching mathematics for relational understanding, make their own decisions, and encourage independence in the children they teach. They are confronted with multiple interpretations of teaching and learning issues and encouraged to appreciate the need to refer to context and personal values when evaluating and making decisions. Students collaborate with the children to develop major integrated mathematics and language projects on topics geared to the children's interests and choices. The above factors provide a great deal of challenge for students.

The quality of the program is directly related to the quality of the pedagogical and interpersonal skills of the lecturers concerned. Although such statements are beginning to recover their status as self-evident, our phenomenological understanding of the connection between teacher quality and program quality has been severely undermined by the theory and research associated with behaviourists of the 'teacher-proof program' school of thought. The more complex the educational program, in fact, the more its success is contingent on the attributes of the lecturers involved (Hill, 1993).

ESTABLISHING THE CONDITIONS FOR CONCEPTUAL CHANGE

An outline follows of how the DIM variables—structure, experiential learning, personalism, and diversity—together with additional strategies tend to bring about the conditions for conceptual change. Establishing the plausibility of change for the students in the present program essentially consists of helping them believe that they are capable of learning mathematics and that the program has the potential to help them. Viewed naively, this appears to be a simple task requiring only straightforward and rational discussion. Attempts to establish plausibility, however, must confront that complex, self-contained, self-fulfilling, self-perpetuating, often irrational but superbly coherent collection of edicts which constitutes the individual's belief system. The degree of discouragement suffered during their past mathematical experiences is such that many students in the present program refuse to believe that it is possible for them to learn mathematics. We encourage students to believe in their ability to learn, but we also believe that such encouragement needs to be reinforced with high expectations. Without such challenge, our belief in students may be interpreted as mere rhetoric. Of the four conditions for conceptual change, the task of helping students change their beliefs is the most difficult to achieve.

One objective in establishing plausibility is to help students become aware that their mathematical incompetence and hatred of mathematics is largely a consequence of past experiences of instrumental—and often poor—teaching. Establishing plausibility involves addressing the issue of the power relations which have resulted in many students who were not successful at school mathematics believing that they are intellectually inferior. The falsehood that only a clever elite have the ability and right to practise mathematics needs to be discussed and debunked (Frankenstein, 1985). This power issue is dealt with partly through our dismissal of the moral element which is often attached to learning mathematics (Skemp, 1989, Buxton, 1981). We openly acknowledge that many students hate mathematics and that they have good reason for hating it, but that nevertheless their aversion need not prevent them from becoming mathematically competent. It is very important, we believe, to demonstrate our acceptance of students' aversion to mathematics. It helps to prove the point that as lecturers we are not demanding that they become like us; rather that we are willing to step into their shoes and see mathematics learning from their perspective (Noddings, 1984). We encourage students to discuss their past mathematics experiences so that they will realise that they are not alone in their concerns about their intellectual capacities. The aim of such discussion is to help students identify with others, and to rationally examine the processes by which they came to hate mathematics, how this deterred them from engaging in mathematical activities, and how they thereby became mathematically incompetent.

The above process is effective only in a personalistic, egalitarian, and respectful classroom environment. Students' views, beliefs, levels of competence, and fears must be accepted; students need to be encouraged to speak their minds without fear of being ridiculed and to understand that they have the opportunity to ask for and accept help. We encourage students to become aware of and dispute their negative self-talk (Munro, 1982; Ellis & Harper, 1975) and to use positive visualisation—to picture themselves doing mathematics well and to say for example: “I may not be able to do this yet, but I am capable of learning how.” Classes are not therapy groups, however, as a highly personalistic climate does not include inappropriate self-disclosure (Knefelkamp, 1981, p. 6). It does include relaxed surroundings and friendly colleagues with whom to experience the pleasure of sharing a common goal and helping each other progress. Working with children in this school-based program also helps to establish the plausibility of learning mathematics because constant feedback from the children ensures that students realise that most of their efforts are successful.

In the school-based program students need to plan new activities every week for the children with whom they work. On the issue of fruitfulness, because, during the weekly sessions, students are not under pressure to perform for assessment purposes, and neither are they expected to control the class—a task which most first-year students dread—to some extent they can enjoy the experience. They are able to experiment, play, and make mistakes. The children they work with enjoy the special attention, look forward to the students' visits, and as a rule are cooperative, friendly, and appreciative. As a result of this interaction most students develop strong feelings of responsibility towards the children they teach. When the students teach well the children demonstrate their pleasure and understanding; the children’s reactions tend to induce reciprocal feelings in the students and a cycle of mutual enjoyment in learning and teaching is initiated—the buzz which sustains many teachers in their work. Learning mathematics and learning how to teach it effectively become worthwhile in such circumstances. Although, for most students, the above experience is predominantly rewarding, it also is intellectually and emotionally demanding and at times disturbing. The program demands that students engage in continual problem-solving. In the school-based setting the relevant problems are immediately apparent to students—problems of learning to teach mathematics, of devising more effective methods than those previously experienced, of forging cooperative relationships with children, of feeling comfortable in the teaching role, and so forth. None of these problems are of immediate relevance to students in a campus-based program. One of the 18-year-old students in the school-based program summed it up: “We're thrown in at the deep-end here. We're constantly faced with different problems to solve. It's really, really hard and it's really good”.

Success in learning mathematics is essential to the establishment of the condition of fruitfulness. We do our best to ensure that students experience mathematical success early in the program. We encourage them to play with ideas and experience the value of creative play in solving problems and developing original ideas. Classwork, homework, examinations, and assignments are designed to promote and test relational understanding rather than instrumental learning. The challenging level of classwork and tests also results in satisfaction when the challenge is met.

The present program is built upon a constructivist philosophy that for students’ learning to be effective they need to construct their own conceptual understanding (von Glasersfeld, 1991). On the issue of intelligibility, because students learn mathematics with relational understanding by using materials, diagrams, and commonsense methods rather than rules and formulae, and apply their new understanding by teaching others, learning mathematics becomes more intelligible. We encourage students to identify with and trust their own commonsense approaches to solving mathematical problems. Only by doing the required work, however, can enduring understanding be gained. It goes without saying that students who do not do the class activities or homework and who do not put sufficient effort into their weekly teaching generally do not pass the subject.

Regarding the establishment of dissatisfaction with existing concepts, mastery-style mini-tests are administered throughout the year in order to help students become aware of and dissatisfied with their current levels of mathematical skills and understandings. A balance must be achieved such that students become enlightened about their mathematical deficiencies but are not unnecessarily discouraged. Also, through working closely with children, students come to understand some of the difficulties encountered by children in struggling to grasp mathematics concepts and the students realise that their old instrumental methods are not equal to the task of teaching children effectively.

The application of the DIM, I have suggested, is likely to foster the conditions for conceptual change identified by Posner et al. (1982). The three DIM variables which provide support for students—structure, experiential learning, and personalism—are designed to establish three of the four conditions for conceptual change—plausibility, intelligibility, and fruitfulness. Evidence from the present study indicates that personalism and experiential learning are strong factors in establishing those three conditions and may be the most significant variables in stimulating productive change. High degrees of experiential learning, personalism, and structure which facilitates rather than restricts, also appear to promote self-control of learning.

Relations between the developmental variables and the conditions for conceptual change are reciprocal and synergystic. The development of trust in fellow classmates (personalism), for example, promotes the belief that the program will be an enabling one (plausibility), which in turn promotes further trust and risk-taking (personalism). One-to-one interaction with a child (experiential) generally results in enhanced satisfaction and independence (fruitfulness and control of learning), thus stimulating further interaction (experiential). The provision of diverse contexts, philosophies, interpretations, and methods for approaching mathematical and teaching/learning processes is intended to provoke dissatisfaction with existing levels of knowledge and understanding. Diversity in the form of unfamiliar arguments by which to dispute negative self-talk also indirectly contributes to the establishment of fruitfulness and plausibility. The manner in which the developmental variables appear to bring about the conditions for conceptual change is outlined in figure 1:

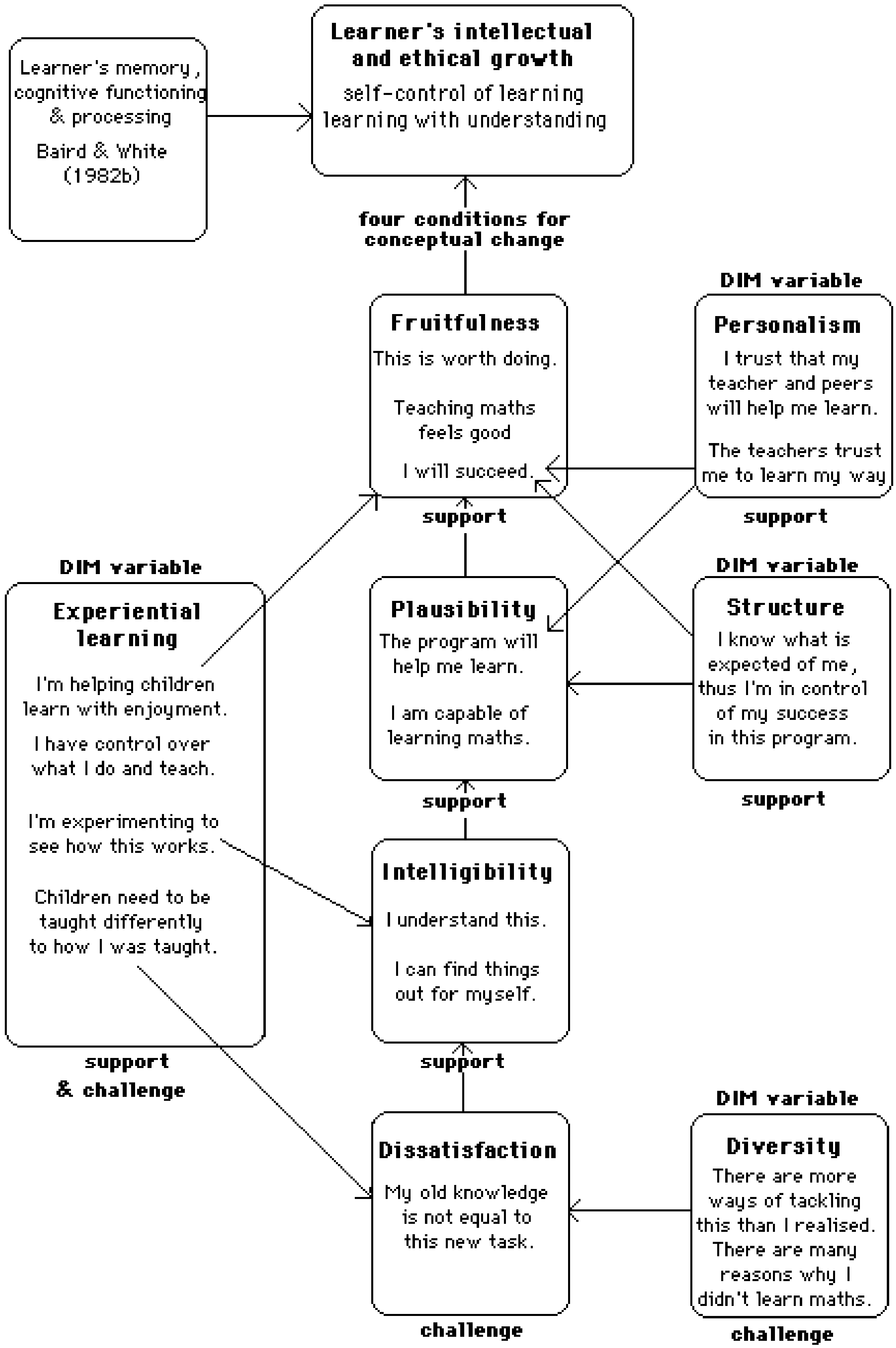

Figure 1 The Developmental Instruction Model and the conditions for conceptual change

In summary, many of the strategies applied in the present program generally may be categorised according to the four DIM variables. In addition to the DIM variables cognitive-change strategies help to establish the plausibility condition regarding the learning and teaching of mathematics; they include cognitive disputation of self-defeating messages and encouragement to develop more positive self-talk such as that listed in Figure 1. Additional strategies include the modelling of appropriate methods for teaching primary children, and discussion of popular falsehoods about mathematics and about teaching and learning.

STUDY DESIGN

The major study on which this paper is based combined quantitative and qualitative approaches, including the use of in-depth interviews, surveys, and statistical analysis of pre- and post- data regarding intellectual development and attitudes towards mathematics. The present paper is concerned with the interview and survey data regarding the establishment of Posner et al.'s conditions of conceptual change. The sample comprised 59 first-year students in the Bachelor of Teaching Primary course on the Toorak campus of Deakin University. In addition, I conducted audiotaped interviews with a randomly selected group of eight students. A ninth student, not randomly selected, was interviewed when one interviewee withdrew from the university before the second interview.

FINDINGS

The students completed a written questionnaire in October 1991. They were asked to describe key factors, if any, which they believed had affected their understanding of mathematics, confidence in themselves as learners of mathematics, and their view of mathematics learning as worthwhile (the conditions of intelligibility, plausibility, and fruitfulness). I did not provide a list of factors from which students could choose because it seemed that this might negate the potential significance of the responses: anyone can choose items from a furnished list without a great deal of thought. The danger in not furnishing a list, however, was that no pattern would be distinguishable among the diversity of the responses.

The data in table 1 indicate the frequency with which particular factors in the program were identified as having contributed to students' beliefs that they had the capacity to learn mathematics (part of the plausibility condition), that mathematics was understandable (intelligibility condition), and that applying effort to learning mathematics was worthwhile (fruitfulness condition). For each of the three conditions listed students were asked to describe the most significant factor which had promoted their learning of mathematics.

Table 1. Most influential single factor spontaneously described as contributing to the conditions of Fruitfulness, Intelligibility, and Plausibility.

Conditions conceptual change n = 59	Teaching children weekly	Hands-on concrete activities	Maths success	Lecturer's encouragement, etc.	Set work, tests, or text	Need qualification	Other, or *Everything*	No response
Fruitful	31%	17%	24%	10%	10%	7%	1%	0%
Intelligible	15%	42%	0%	24%	10%	0%	5%	4%
Plausible	15%	15%	29%	29%	3%	0%	5%	4%

Given the spontaneous nature of the responses it is interesting that they fell almost universally into the first five categories listed across the table. For each of Posner et al.'s three conditions the vast majority of responses fell into three or four main categories. Because 83% of the students judged both the program text and tests as ‘very useful’ or ‘mostly useful’, the relatively infrequent mention of tests and manual as key factors does not indicate a lack of appreciation of their worth; it indicates that other factors were more significant. Questionnaire responses during the past five years have suggested that the mathematics program generally is held in high esteem. In 1991 and 1992 the relevance of the program was attested to by 95% of the students surveyed. Clearly, the factors identified above are not chosen because they are the only highpoints in an otherwise featureless landscape. Students’ criticisms of the present mathematics program tend to focus on factors which we as staff also criticise and find difficult to manage—the lack of time for the establishment of new understandings, and so forth. No boundaries exist between the conditions for conceptual change; like the DIM variables, they overlap. The interview data suggest that particular variables affect more than one condition.

On the issue of fruitfulness, the question asked was: “Which aspect of the course most helped to motivate you to learn mathematics, that is, what, if anything, made it worthwhile to put effort into?” 48% of students described one or other of the factors comprising the experiential component of the program: teaching children, and using concrete materials, for example: “The fact that we had real live kids to teach was the real motivation behind grasping all the maths!”; “The work with the kids of course!!” 24% identified the second-order component ‘success’ as the key ingredient. Clearly, the degree of success achieved is a result of the operation of other factors. Success does not materialise without effort, encouragement, and appropriate opportunities to understand and practise the task at hand. Students are unlikely to put in the effort required to achieve success if they believe they cannot understand mathematics—to be able to attribute one’s failure to a lack of effort than to a lack of ability is less of a blow to one’s self-esteem (Weiner, 1980). It is likely, I suggest, that the experiential component of the program contributed significantly to the achievement of success for many students. In answer to the question of what made it worthwhile for them to put effort into learning mathematics (a subject disliked by all but one), the interviewees also focussed on the need to teach well, the class activities, and the desire for success. Some also spoke of how peers and their lecturer influenced their understanding, enjoyment, and motivation:

What made it worthwhile this year—putting any effort in?

> The fact that I want to be able to teach the subject well made it worthwhile for me to put the effort in, so the end result made the working worthwhile.

> In this course...I'm motivated to learn it because I know I'll use it a lot...and it's all going to be relevant because...everything we learn we're going to teach.

> *Now* I know I went on with [maths] for one reason: that was the fact that I've had so much trouble with it, that I'll probably be a good teacher of it because I can understand why the kids have problems with it. I think that's really helpful actually: not being a wizz at maths to begin with....the other thing that made me keep going was the encouragement I had from you and other students too....and the fact is that I *do* want to be a teacher.

> It's got a lot to do with you—I mean the way you're teaching it....If it's fun...it takes away the fear and anxiety—you're more relaxed, you're at ease with it....I really want to understand it so that I can be able to teach it.

These findings indicate, I suggest, that the condition of fruitfulness largely depends upon one's personal purpose for learning and one's degree of control over the process of fulfilling that purpose. The students' purposes for learning fell into three main categories: to be able to teach mathematics to children, to understand mathematics, to succeed in the subject. From the questionnaire and interview responses it appears that the program was in tune with these goals and enabled students to maintain a considerable degree of control over the process of achieving them.

Regarding the establishment of intelligibility, the question asked of students was: “Which aspect of the course most helped you to understand mathematics?” 57% responded that mathematics was made intelligible for them through teaching or through the use of concrete materials. Clearly the act of teaching something can be an effective vehicle for clarifying one’s own level of understanding of the topic. Their lecturer’s encouragement was identified by a further quarter as a major factor. It is not clear to me how the logic works here. Perhaps the teacher’s encouragement is what kept them trying. When questioned about what had helped to make mathematics understandable, the interviewees focussed almost entirely on the importance of using concrete materials and mathematical situations with clear connections to everyday life.

> The constant use of materials—that always helps clarify things in my tiny brain. I think the most valuable part is the initial tackling of a new problem...the explanations and working as a group in the classroom and working through problems...constantly speaking about them. Certainly the things that you said about—translate it into plain English that a six year old could understand—that sort of thing is really valuable...because you can start not looking at the sum as just a sum, but as a piece of logic to be ingested and worked out....When you're doing maths in school often just knowing how it happens is enough—not knowing why. And so I didn't realise that there would be all this questioning of how things work, why they work...In that way I find maths very difficult because I can often do the things, but...if you ask me “Why?” I'd say "I don't know"...that's the hardest part I think—really truly understanding how it happens.

> You're showing us...that it's meaningful and...it's used every day....When the blocks come out it's simpler, you can relate it to

the sum...because there's something physical there that's showing you....On...a sheet it's just numbers—it's just a blur....I was amazed when I first saw the blocks...I thought they were brilliant. I just thought "Oh why didn't I have them...it might have been easier."...I'm still finding it difficult now, but I want to understand it because...how am I going to teach it if I don't understand it?...If I've got insecurities about it...I'm going to pass them to the kids—they're going to know if I don't understand it.

I think that was vital to have those little blockies....when...I started learning...I realised that...I could not understand it unless I started off either drawing it or using the blockies...the only way I could do it was to use the concrete materials, and I've never ever had that problem before.

The concrete materials that you use with us—for example finding out the area using the little squares...I wouldn't have a clue how to do it first—but that helps.

I think...the need for...concrete materials...and teaching that and really hammering that home, has been really good because it's not only helped the kids learn maths, it's helped us learn maths...therefore it makes it easier to teach too because you understand it better and you can explain it to the kids—it's not "That's the way it is". *Why does it help?* 'Cause you can do it, you can actually sort of make it...feel it, see it. It's not just going through your mind "Oh oh"...and you have to try and picture it...it's actually there in front of you and you can touch it...which just makes you understand it better, rather than trying to put this picture in your mind, trying to imagine...these abstract concepts. That's been really good....I thought that was good from the start...if that method was more...into secondary college as well I'm sure...a lot more kids would be doing maths.

That thing we were doing the other day with the tiles—I was looking for the sum, you know, and I didn't think of the

> common sense...logical approach to it, I just went on this tangent looking for sums....I instantly started writing down numbers instead of drawing a picture and estimating how many tiles would fit along one side...which once I saw how you could do that, I mean, it was so much easier and made much more sense too—that's how you would do it, you know....maths, when it's on paper doesn't seem real...it's just numbers...that's the way they've been taught...to find the sum in it and write it down...instead of...having it made real to them by being put in actual situations where they're going to have to use maths....A lot of teachers...they probably think "Oh it's just quicker to run through this on the board". The penny dropped for me...because I'm a bit rusty on maths, usually I do that sort of problem with numbers on the page, but once that started to stuff up I thought "What's going on here?" and then once I did it practically, well it was like, you know, "That's easy"....Once you do it practically, after a while there's no need to actually cut out the tiles, you can do it just on paper—you know what you're talking about.

Regarding the establishment of plausibility, the question asked was: “Which aspect of the course most helped you to believe in your ability to learn mathematics?” Three factors were identified in approximately equal measure: experiential learning (30%), personalism (29%), and success (29%). As mentioned earlier, the second-order factor—success—probably results from one or more of the other factors. The question as it was asked is likely to elicit comments about success. Clearly, belief in one’s capability stems from evidence of that capability, that is, success. Responses to a second question: “To what do you attribute your success?” may have provoked more enlightening comments.

The nature of what we say to ourselves, according to cognitive-behavioural psychologists, significantly influences our likelihood of success in learning (part of the plausibility condition). Even given adequate support from peers and teachers, if we continue to tell ourselves how hopeless we are at mathematics, failure almost automatically follows because we will not be likely to bring ourselves to exert the effort required to succeed. The interviewees spoke about the negative self-talk in which they had engaged throughout their schooling. Often, internal disputes occur but the goodies do not always win:

> As soon as I can't do something like a sum I think "That's it, I can't do maths, forget it" and I just put it away and go and find something that I can do....It's...a feeling of defeat—just feel like maths has got the better of you and no hope for you anymore, you're not a good person anymore....If they're smart, they're good. *And in the moral sense are you saying?* Yes....you sort of relate the two—and you see people who aren't good at maths who weren't good people...they're really sort of rude and...won't do what the teacher says in class. *If you're good at English are you also...morally good?* I think in a way it is seen as that, but not so much as with maths, because I think people think maths is something harder to learn. Until I can understand it I'll be down in the dumps about it, and if I'm that way then I don't do anything about it, I just won't try it—"What's the point, I can't do this".

> “Oh forget it then. It's the second time and I still don't know what it says....panic...I won't get it done in time and I won't pass”.

> I have still got this incredible mental block...when I look at maths questions its like a mental stage fright and...I don't know how to...change it...I've got this incredible thing in my mind saying "No, you've spent 30 odd years not knowing—why the heck should you suddenly—you're not meant to"....I think a lot of it's subconscious.... [Consciously I say] "I'll have a go" and even if I'm right that's a real shock, I can't believe I've actually got something right, or if I'm wrong I hear myself saying "Well see...you knew you wouldn't be able to do it"....it's like learning a foreign language and I don't know the rules. I still get panicky

because I don't expect to be right, I expect to be wrong....I know what I say, I say "God, what if I'm wrong when I've got this concrete stuff in front of me—there'll be no hope if I get it wrong when I'm using that".

[Having to do maths] was one reason that I almost wasn't going to do the course....I've wanted to do primary teaching...it's still in the back of my mind that I will fail the course because I'm doing maths...if it wasn't for having to do maths I would probably pass this course...it sounds very negative doesn't it...when I think that...it's more matter of fact to myself because I'm so used for so long to assuming that I can't do it that that's how it is...without being melodramatic that is just how it is....I get really enthused after the class on Friday...by the middle of the week I'm back to thinking "Oh well"....It's like wanting to walk on water, for me....That's what I keep telling myself "I want to pass, I do, but it will be a miracle if I do", yeah that's absolutely it in a nutshell, that's what I tell myself all the time.

Having been someone who felt like they'd failed maths...how did you change? How did you become somebody who succeeded and drove away that fear that you had originally? Sheer persistance, persisted, and haggled with myself. Maths multiplies your choices. With me it was a great exhibition of self-will...one bit of yourself saying “No just drop it, it's too hard, it's easier to drop it, go and do something else, do something that you like doing”—we're talking about high school—“Stuff maths, you don't need it” and the other part of yourself saying “Hang on, yeah, it's a load of shit but I can do it, I showed that I can do it by passing that year 10 exam, and you just need a bit of determination, persistence”, and in the end that side of me won out.

Anecdotal and interview data suggest that, of all primary mathematics topics, fractions appear to rate highest on the scale of loathing. Vulgar fractions appear to have struck more fear into more hearts—more so than decimals which probably would rate a close second. The reason appears to be that the mathematical rules for performing operations on fractions and decimals were taught bereft of meaning: “turn it upside down and multiply”, for example. What kind of instruction is that? One guaranteed to cause large numbers of children seriously to doubt their cognitive capacity to deal with mathematics and in many cases to turn them off learning mathematics for life. Unfortunately it is still the case that few teachers and few textbooks explain the logical processes which underlie the mathematical rules. It is clear from the comments of the interviewees and many other students that their understanding of fraction concepts stagnated when rules for equivalent fractions were introduced to them. Worse, some students have never connected their informal understanding of cutting a pizza into six pieces and taking one piece with the mathematical symbol $^1/_6$.

The interviewees were asked about factors in the present course which had influenced their confidence in their capability to learn mathematics. They referred to the encouragement of peers and lecturer—the importance of knowing that others believe in your capability—and the realisation that they were not alone in feeling anxious about mathematics:

> Seeing that I'm not the only...one that was hating it....I realised how many people hated maths...couldn't believe it...the whole class.

> It's got to be a friendly and supportive environment—you can't be afraid to ask questions—and the people...be willing to help you...when you don't get it....That's what we've got at the moment in our class.

> I understand the things we are doing in class more...now I like to get involved....If you want to learn something you've got to go into the classroom saying "Oh this is an excellent class, I really love the stuff we're doing." If you don't have this attitude

> then...you can't learn. Even if you might know the stuff, you don't want to learn because you don't like your teacher or you don't like the people in the class or the atmosphere is really shitty....It does help with a little bit of humour—from the teacher as well—so we relax a bit.
>
> I think it's important that a teacher shows that they care about the students in a genuine way. You don't feel as threatened when the teacher is friendly towards you, so feel confident enough to ask for help when needed. When a teacher treats you as an equal, especially in your senior years, you become more willing to learn from them. It is no use trying to teach me anything unless I want to learn....It is important that you feel comfortable with the people in your class or you may be too scared to ask for help. If you are in a position where you are too scared to ask for help, you fall behind because you don't understand and then you start to feel like you are no good—[a] defeated feeling. You also become anxious about the class and panic every time you get to that time of the week. This sort of fear prevents you from learning all that you are capable of.
>
> If they felt [anxious] because of their prior experiences of maths, I think the best thing that can be done is what you've been doing—and what you did especially at the start of the year—which was just open up and show yourself as another vunerable person as well, who's had problems with maths and...like what I discovered in year-12, my maths teacher didn't know everything, and...no maths teachers know everything.

In summary, the survey and interview data indicate that encouragement from others, success, and the construction of mathematical schemata through concrete learning and teaching situations are significant factors in the establishment of plausibility. Most of the interviewees identified a supportive classroom environment as significant—one in which they trusted they would be treated with care and would not be disparaged. “Having other people believe in you”, clearly, encourages you to believe in yourself.

Regarding Posner et al.’s condition of dissatisfaction: the questionnaire asked students to rate the extent to which they realised that they needed to learn to teach mathematics differently from how they had been taught at school. 88% indicated they had developed a strong awareness of this need; another 10% said their level of awareness was ‘average’. Thus, by the end of the present mathematics program, there was virtually unanimous acknowledgement among students of the need to adopt new ways of teaching mathematics. Because I did not collect initial data regarding this issue, no comparison is possible. On anecdotal and interview evidence, however, the postdata indicate that a substantial change occurred. That the majority of students rated their past experience of mathematics schooling as, at best, unsatisfactory, both for them as learners and as a model for teaching children, is supported by the above data. The interviewees commented on the factors which had encouraged them to feel dissatisfied with their current level of mathematical understanding and skills and their notions about how mathematics should be taught:

> The first test proved how bad I was at maths, and...working with the kids...always helps you recognise when you need to learn more....It's...like when you know how to read—you can't imagine not being able to read...it happens with maths—once you know it you can't imagine not knowing it, so...I find that very difficult. If my kids say "Why?" they stump me so often—"Well I don't know—it just does."

> You gave us that test and I suddenly realised that things were going to be a little more difficult than I'd thought—Oh dear—yes, that was a bit of a shock....Yes, of great value...a bit of a downer...it just helped me see where my incredible gaps were....Whereas when I came into college I thought "Oh yeah...I can probably do this O.K."...as it's turned out, I need to put a lot

more time into it than I originally thought. So that test was valuable in showing the gaps.

And seeing...kids in the MALL [prep grade]...realising that they've already got those feelings [hating maths]....I think they've got to be shown that it's useful every day, like we're trying to do with them, and like you do with us.

You need to see how it *works*...like you might think these games are a bit [mildly derogatory gesture], but then you see the kids doing them and liking them and...it's hard to imagine the problems they're going to have with it. You just think "Oh that's straightforward", and then you see...they just don't understand and you...think "Oh...yeah"—you need to work out what language to use and everything which is important.

The fact that I had trouble with it in my own learning...so...something else needs to be taught a different way...so the kids don't have to go through what I went through.

I didn't think that primary teaching would be a doddle but it [MALL] really does show you how demanding—if you want to do it properly—it is...because you've got all these kids with all these different abilities and some of them don't want to be at school...and it's hard because you can't be in 30 little heads at the one time.

Anecdotally students relate that several factors contribute to a feeling of dissatisfaction with their current knowledge about mathematics and mathematics teaching. This anecdotal evidence is supported by the interview data. The factors are: the program tests; realising that they do not understand *why* mathematics works; and experiencing at first hand children's confusion about mathematics. The latter factor has the greatest impact on those students who are committed to teaching: clearly they develop considerable empathy with the children with whom they work. All the interviewees—bar one who does not want to teach—emphasised that their work with the children was instrumental in developing their awareness that they needed to develop new ways of teaching mathematics.

Although structure components (program requirements, tests, homework, text, etc.) were identified less frequently, clearly the existence of these supports improves the chances of students achieving success. While these structure components fulfil their role of clarifying and monitoring the process of achievement, their presence may be relatively unobtrusive.

The results suggest that, as expected, the two DIM variables, experiential learning and personalism, support the establishment of the conditions of plausibility, intelligibility, and fruitfulness. Successful performance in tests and so forth is identified as an important factor promoting the conditions of plausibility and fruitfulness, although, clearly, success is related to fundamental factors such as having the opportunity, via the use of concrete materials, for example, to understand the mathematical ideas being presented.

DISCUSSION AND CONCLUSIONS

Posner et al. identify and describe the nature of the necessary conditions for accommodatory change. How these conditions might be created is problematic and, clearly, the manner in which teachers attempt to establish them will be related to their particular context. Although the following notion is not addressed by Posner et al., the establishment of the condition of fruitfulness of accommodating new ideas depends on the degree of the learner's control over the learning and the degree to which the adoption of the new ideas fulfils the learner's personal goals. It is our task as teachers to enable this connection to be made by students and to maintain its viability.

As it is structured, the present school-based program encourages most students to take responsibility for their mathematics teaching and, as a necessary concomitant, for their mathematics learning. Three objectives of the program thus are addressed immediately: concordance with students' goals; students' self-control of their mathematics learning; and students' commitment to relational learning. It almost goes without saying that the degree to which students make use of the opportunities available in the program varies directly with the degree of their commitment to teaching as a career. Students who are not interested in teaching remain relatively unmoved by the program. In general the teaching component of the program is valued highly; the responsibility, however, clearly is initially onerous—even for highly committed students. Our insistance that students adopt more democratic approaches to teaching than those familiar to them provokes confusion and anxiety in many students. They have faith in didactic, controlling methods, yet we ask them not to adopt such teaching roles. We suggest that they make friends with children, yet they believe that being friendly will deter children from being respectful and compliant towards them. And, in the main, such beliefs are reinforced by their parents, peers, school teachers, some children, and some lecturers. Also, the teaching role itself is unfamiliar to students—just a few weeks ago they were school students. Worst of all, most students do not understand or like mathematics; the thought of teaching it, therefore, is distasteful and often provokes anxiety and avoidance behaviour.

What encourages students to persevere? Several factors support the process. First, the personal rewards are substantial. Students' efforts are rewarded by the pleasure in seeing their children learn and develop in optimal conditions of personal engagement between student and child. The children respond with pleasure both to students' care and respect for them and to students' provision of meaningful, relevant, and enjoyable learning experiences in which the children's input is sought and welcomed. The two twelve-week periods with children enable the establishment of strong personal relationships; relationships which are of special benefit to children preoccupied with personal, social, or learning difficulties.

Second, the group of children assigned to each student is small—two or three at the most initially—so that issues about who is in control do not assume magnified importance. This, interestingly, provokes disquiet among a substantial number of students who wonder whether such a program bears much relevance to teaching. The fact that they are not expected to control a class is interpreted by some as undermining the value of the experience because, to these students, teaching *is* controlling. What they teach and how the children learn are viewed as unproblematic because the curriculum exists, the teacher follows it, and the children learn it. Simple. The only difficult thing, it follows, is the maintenance of order and discipline while they get on with it. In order to nip in the bud any accusations of irrelevance, we need to explain and reiterate to students that we want them to concentrate on understanding how individual children learn—their preferred learning styles, needs, interests, problems, likes and dislikes, eccentricities, and so forth—and how to cater for these in ways that enhance the children's and their own lives. We assure students that they have plenty of time to learn how to manage a whole class and that their present experience can only contribute to a successful outcome in that regard. A few, of course, continue to despair at our ivory-tower ignorance of what constitutes the real issues for those at the chalkface.

The third source of support for students in their teaching role is that, during students' teaching sessions, lecturers and classroom teachers primarily act as collaborators, not as assessors. Gradually, taking risks and making mistakes are recognised by students as valuable—albeit painful—learning opportunities; they are congratulated for making mistakes, and particularly congratulated for learning from them. In such a situation students' development as learners and as teachers is accelerated markedly in accordance with their degree of risk-taking. Fourth, because, every week during the academic year, lecturers and teachers are present with students while they teach, they can perceive immediately students' needs and offer support for as long as it is required.

According to psychodynamic theory, it is the emotional component of learning, stemming from powerful emotional experiences, which provokes behavioural change (Goldfried, 1979; Epstein, 1980). Challenging experiences, depending on their intensity and on students' readiness, may spur either development or retreat (Perry, 1981). With lecturers providing some wisdom of hindsight, and in the company of peers likewise motivated to understand their experiences, the opportunity to discuss and reflect on those experiences is more likely to tip the balance in the direction of development. The present findings indicate that productive change in learning and teaching practices is more likely to flow from emotionally powerful experiences in which theory, practice, and reflection are integrated consistently and repeatedly. Student teachers need to practise teaching, experience the responses of children to their teaching, and discuss and reflect on these experiences over a substantial period of time.

If anything formed the crux of the present study it was the question of how to encourage students to value relational learning in mathematics. As a result of their experiences of mathematics schooling, however, many students lacked confidence in their ability to learn mathematics and believed that their only chance of success was to memorise the correct rules. Instrumental learning constitutes the normal school mode and it is what most students have grown to accept—worse—to want. We, the lecturers in the present program, knew how to teach for relational learning, and our rationale was clear; when students did not agree, however, our efforts largely went to waste. Why, when it has never been an option for them, should students value relational learning in mathematics?

Interview, survey, and anecdotal evidence suggests that, while many students came to value relational learning because it enabled them to understand and gain control over mathematical processes in a way they had not experienced previously, others valued it because they experienced intellectually and emotionally children's need and desire to learn in that manner. By no means was this experience immediately, automatically, and consciously appraised as such. That relational learning is significant was realised and clarified through reflection on, and discussion of, their experiences. Only then could students see that they could not fulfil children's needs unless they taught in a way that fostered relational learning, and that they could not teach in this way until they had also learnt in that way.

The condition of fruitfulness, that is, the degree to which it is felt worthwhile to put effort into learning, depends on the extent to which we are able to fulfil our goals and the degree of control we have over that process. The primary goal of most students is to teach well. They need to have control over the process of reaching this goal. When their attempts to reach their objective are hindered by their being required to spend time engaging in activities which they consider to be irrelevant to their purposes, they swiftly lose interest in the enterprise. The corollary is that they tend to applaud and embrace opportunities which enable them to focus on achieving their goals.

In summary, relational learning was valued by students in the present course when they made a connection between it and their personal goals—when they recognised that relational learning would contribute to the achievement of their goals. As long as that connection is made and as long as students maintain control over the process of achieving their goals, they will value and put effort into relational learning. And now I do not feel, as I often did before the establishment of the school-based program, that much of the effort I put into teaching is wasted.

REFERENCES

Anderson, A., Crawford, K. & Sinclair, K. (1989, July). Basic numeracy skills of primary education student teachers at Sydney University and St George Institute of Education. Paper delivered at the the 12th annual conference of the Mathematics Education Research Group of Australasia, Bathurst, NSW.

Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 10,* 2, 10-16.

Batten, M., Griffin, M. & Ainley, J. (1991). *Recently recruited teachers: their views and experiences of pre-service education, professional development and teaching*. Canberra: Australian Government Publishing Service.

Bennett, N., Desforges, C., Cockburn, A. & Wilkinson, B. (1984). *The quality of pupil learning experiences.* London: Erlbaum.

Bishop, A. (1988). *Mathematical enculturation*. Dordrecht, Holland: Kluwer Academic Publishers.

Borko, H., Eisenhart, M., Brown, C., Underhill, R., Jones, D. & Agard, P. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23,* 3, 194-222.

Buxton, L. (1981). *Do you panic about maths?* London: Heinemann.

Clements, M. A. (1989). *Mathematics for the minority*. Geelong: Deakin University Press.

Davis, C. (1985). A teacher's dilemma. *For the Learning of Mathematics, 5*, 3, 24-26.

Davis, R. B. (1984). *Learning mathematics: the cognitive science approach to mathematics education*. London: Croom Helm.

Dettrick, G. W. (1981). Mathematics in-service guidelines study: an interim report. Unpublished paper. Gippsland Institute of Advanced Education.

Ellis, A. & Harper, R. A. (1975). *A new guide to rational living*. California: Wiltshire Book Co.

Epstein, S. (1980). Self-concept: A review and the proposal of an integrated theory of personality. In E. Staub (Ed.), *Personality:*

basic issues and current research. Englewood Cliffs, New Jersey: Prentice-Hall.

Evans, G. (1987). Teacher education in the 70s and 80s. In K. J. Eltis (Ed.), *Australian Teacher Education in Review*. South Australia: South Pacific Association for Teacher Education Inc.

Frankenstein, M. (1985). A different third R: Radical math. Unpublished doctoral dissertation, University of Maryland.

Freeman, V. (1986). Anxiety reduction course. Unpublished manuscript. Melbourne: Melbourne College of Textiles.

Fullerton, T. (1992). Micro-skills or micro-tasks? *South Pacific Journal of Teacher Education, 20*, 1, 25-34.

Goldfried, M. R. (1979). Anxiety reduction through cognitive-behavioural intervention. In P. C. Kendall & S. D. Hollon (Eds.), *Cognitive-behavioural interventions: theory, research, and procedures*. New York: Academic Press.

Handal, G. & Lauvas, P. (1987). *Promoting reflective teaching*. London: Open University Press.

Hill, L. M. (1993). Promoting self-control of mathematics learning for pre-service primary teachers. Doctoral thesis in preparation. Melbourne: Monash University.

Howard, P. (1989, July). Student teachers' attitudes and perceptions towards mathematics. Paper presented at the the 12th annual conference of the Mathematics Education Research Group of Australasia, Bathurst, NSW.

Kline, M. (1977). *Why the professor can't teach: Mathematics and the dilemma of university education*. New York: St. Martin's Press.

Knefelkamp, L. L. (1981). Developmental Instruction. Unpublished manuscript, University of Maryland.

Munro, J. (1982). Coping with mathematics anxiety. Unpublished manuscript, Mathematics Learning Centre, Victoria College (now Deakin University), Melbourne.

Noddings, N. (1984). *Caring*. Berkeley, California: University of California Press.

Pateman, N. A. (1989). *Teaching mathematics - a tantalising enterprise*. Geelong: Deakin University Press.

Perry, W. (1981). Cognitive and ethical growth: the making of meaning. In A. Chickering (Ed.), *The modern American college: responding to the new realities of diverse students and a changing society*. San Francisco: Jossey Bass.

Perry, W. (1988). Different worlds in the same classroom. In P. Ramsden (Ed.), *Improving learning: New Perspectives*. London: Kogan Page.

Posner, G. J., Strike, K. A., Hewson, P. W. & Gertzog, W. A. (1982). Accommodation of a scientific conception: Toward a theory of conceptual change. *Science Education*, *66*, 211-227.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: the disasters of "well-taught" mathematics courses. *Educational Psychologist, 23*, 145-166.

Shearn, E. L. & Davidson, N. (1989, March). Use of small-group teaching and cognitive developmental instruction in a mathematical course for prospective elementary school teachers. Paper presented at the annual meeting of the American Education Research Association, San Francisco.

Sirotnik, K. A. (1983). What you see is what you get: Consistency, persistency and mediocrity in classrooms. *Harvard Educational Review*, *53*, 1, 16-31.

Skemp, R. (1975). *The psychology of learning mathematics*. Harmondsworth, Middlesex: Penguin.

Skemp, R. (1979). *Intelligence, learning and action*. Chichester: John Wiley.

Skemp, R. (1989). *Mathematics in the primary school*. London: Routledge.

Speedy, G., Fensham, P. J. & Annise, C. (1989). *Report of the discipline review of teacher education in mathematics and science* (Vol. 1). Canberra: Australian Government Publishing Service.

Sullivan, P. (1987). The impact of a pre-service mathematics education course on beginning primary teachers. *Research in Mathematics Education in Australia,* August, 1-9.

Sweller, J. & Low, R. (1992). Some cognitive factors relevant to mathematics instruction. *Mathematics Education Research Journal, 4*, 1, 83-94.

Tanner, H. (1989). Introducing investigations. *Mathematics Teaching, 127,* 20-23.

Tisher, R. P. & Klinzing, H. (1992). Procedures to develop classroom teaching skills: Modelling, cases, simulations and micro-teaching. *South Pacific Journal of Teacher Education, 20*, 1, 35-48.

von Glasersfeld, E. (1991). Introduction. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. xiii-xx). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Watson, J. M. (1987). The attitudes of pre-service primary teachers towards mathematics: some observations. *Research in Mathematics Education in Australia,* August, 48-56.

Weiner, B. (1980). *Human motivation*. San Francisco: Holt, Rinehart and Winston.

White, R. T. (1988). *Learning science*. London: Blackwell.

Zahorik, J. A. (1990). Stability and flexibility in teaching. *Teaching and Teacher Education, 6*, 1, 69-80.

Ziukelis, R. (1990, July). Assessing Mathematical Competency of Primary Teaching Education Students. Paper presented at the annual conference of the Mathematics Education Research Group of Australia, Hobart.