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SCHEMATIC MODEL FOR CATEGORIZING CHILDREN'S ERRORS IN MATHEMATICS

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Abstract

The paper is concerned with the analysis of children's errors in solving mathematical problems. Previous studies in error analysis were reviewed and errors were synthesized which result in a form of deficient or erroneous knowledge network: language, operational, mathematical thematic and psychological types of knowledge. The present study looks into the types of errors made by pupils in the process of solving problems. A schematic model describing the errors made is developed which comprises two levels. The first level is categorized in terms of strategic schemas. With respect to this, five categories of errors are identified: no solution, using irrelevant procedure, incomplete schema with no errors, incomplete schema with errors and complete schema but with errors. The second level is categorized in accordance with the classical ways of classifying errors. The second level is thought to be subsumed under the first level of errors. The paper describes the procedures and the methods which lead to the development of the model on the basis of pupils' schematic knowledge in solving a ratio and proportion problem.

Schematic Model for Categorizing Children's Errors in Mathematics

Review of Previous Studies on Error Analysis

Error analysis, like problem solving, has been a major theme of research for a long time in mathematics education. A seminal study by Roberts (1968) sparked off this area of investigation. To date, the topic of error analysis has involved not only the identification of pupils' errors but also their thinking processes which lead to mathematical difficulties (Rees and Barr, 1984). A study of previous research on error analysis shows that error analysis research centres around computation and word problems. Thus the following paragraphs review previous research which are classified under these two headings.

Computation Problems

For the computation problems, Robert (1968), Engelhardt (1977) and Brown and Burton (1978) made great contribution to the analysis of pupils' errors. Roberts classified four major categories of failure strategies: wrong operation, obvious computation error, defective algorithm and random responses. Engelhardt (1977) identified nine types of errors: basic facts, defective algorithm, incomplete algorithm, incorrect operation, inappropriate operation, inappropriate inversion, grouping, identity and zero errors. Engelhardt (Spiers, 1987) commented that children who consistently commit the same types of errors were probably not careless but these errors are a reflection of children's cognitive styles or their stage of development. Brown and Burton's (1978) approach of identifying errors is probably influenced by the access to advanced technology with the use of computer. For Brown and Burton, a large number of systematic errors in subtraction is identified through the use of a computer program called "Buggy". They suggested that children might apply consistently a procedural rule containing one or more small bugs which produce errors on many other occasions.

Word Problems

The second group of researchers on error analysis is concerned with word problems. In Australia, the seminal study of Newman on diagnostic model for identifying errors has led to a series of follow-up studies by Watson (1980), Clements (1982), Clarkson (1980) and Casey (1980). Jones (1984) summarized and tabulated the results of studies carried out by Watson, Clarkson and Newman using Newman's error categories. In the model, there is a hierarchy of performance strategies that need to be applied to solve written mathematical tasks successfully. Eight error categories were classified that corresponded to the performance strategies. These categories are (a) reading ability, (b) comprehension, (c) transformation, (d) process skills, (e) encoding, (f) careless error, (g) motivation and (h) question form. In Newman's definition, transformation refers to the activity carried out to convert the words to the selection of appropriate mathematical model.

Table 1 shows the percentage of errors for each Newman Error Category (abstracted from Jones, 1984).

Table 1 : Percentages of Errors for Each Newman Error Category

Investigators	Error Categories							
	a	b	c	d	e	f	g	h
Newman 13	22	12	26	2	22	3	0	
Watson 15	26	7	47	3	2	0	0	
Clements	5	8	25	32	2	28	0	0
Clarkson	12	21	23	31	1	12	0	0

a - reading ability e - encoding
b - comprehension f - careless error
c - transformation g - motivation
d - process skills h - question form

Newman's result shows that 35% (categories a and b) of the total errors made by the subjects occurred in reading and comprehending the problem before they could even apply mathematical knowledge to attempt the problem. Her result is supported by Clarkson (shown by the figures from the table). The above studies also indicate that a very high % of the subjects made errors in process skills (under category d from Table 1). However, Clement's study suggested that recall and comprehension difficulties caused fewer errors but transformation and process skills errors were high.

Studies on word problems' errors are also carried out outside Australia such as Radatz (1979), De Corte, E and Verschaffel, L (1989). Radatz (1979) proposed an information processing classification of errors as follows: (a) language difficulties, (b) difficulties in obtaining spatial information, (c) deficient mastery of prerequisite facts and concepts, (d) incorrect associations or rigidity of thinking and (e) application of irrelevant rules or strategies. According to Radatz, like Newman, a misunderstanding of the semantics of mathematical text is often the main source of pupils' errors. Spatial difficulty is another type of difficulty identified as a result of the Bruner's movement on representation models in iconic instructions, diagrams and visualization of the conditions in mathematical texts. One factor that Radatz has included is the pupils' negative transfer as a result of rigidity of thinking. This can be explained by pupils' tendency to develop cognitive operations and continue to use them even the fundamental conditions of the mathematical task have changed.

Summary of Different Types of Errors Identified from the Previous Studies

The review presented in the previous paragraphs examines research that focuses on two categories of problems: the computation problem and the word problem. Each study produces its model of errors types. However, they do have some common categories of errors among them. Observations of the various types of errors identified by the previous researchers shown in the previous paragraphs show that they can be summarised into a network of categorial errors made by problem solvers. They are classified in terms of four groups of knowledge: language, mathematics thematic, operational and psychological. The operational knowledge is further subdivided into external and internal knowledge. The different categories of errors are summarized in the Figure 1 below:

Figure 1: Summary of the Types of Errors Obtained from Various Studies on Error Analysis

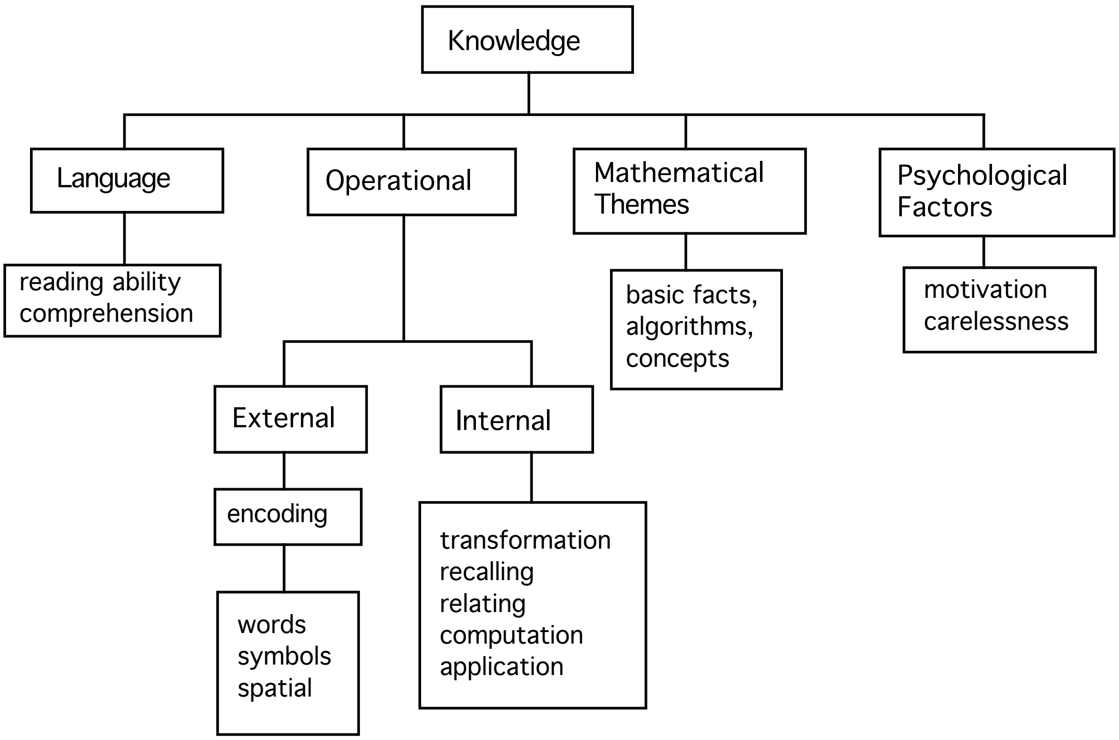


Figure 1 shows that the reading ability and comprehension are subsumed under language. Errors which are due to children's inability to read word problems and/or to understand the problem are classified under this category. Understanding the problem includes interpretation of mathematical language. Under the operational knowledge, the external operation is concerned with encoding of the various types of information such as verbal statements, symbols and spatial figures. Errors which involve encoding words, symbols or spatial features in an incorrect form are classified under this category. They are classified as external operational knowledge because the inforamtion is retrieved from external source i.e. the question. For the internal operation, some activities identified are transformation, recalling, relating computation and application of information. These activities are classified as internal operational knowledge because they are operated within the short-term memory of the brain. The mathematical themes include basic facts, algorithms and concepts. The psychological knowledge involves motivation and carelessness in problem solvers' written work.

Schematic Approach for Explaining Pupils' Strategies in Solving Word Problems

Schematic Theory

Silver (1982) raised the issue of schematic knowledge in educational research. He felt that more research has to be directed towards the content and organization of problem solvers' knowledge. The area of study has to be related to psychologists' research such as the influence of schemas to knowledge organization.

Thorndyke and Hayes-Roth (1979) defined schema as a cluster of knowledge of concepts and associations between concepts. Its structures are assumed to be shared by multiple representations of information from diverse contexts. Rumehardts and Orthorny (1977) also defined schema in a similar way in terms of concepts and their relationships. Schemas are data structures that represent the generic concepts stored in memory. A schema contains the network of interrelationships between different sets of knowledge that constitute a concept.

A Study to Analyze Pupils' Strategies in Solving a Ratio and ProportionProblem

The schematic theory provides the basis for analyzing children's correct and incorrect solutions in the study. The study is concerned with administering a problem from the topic ratio and proportions to pupils. Their solutions were examined carefully to identify the types of knowledge used and the causes of errors were determined. A full report is described below.

Instrument

A ratio and proportion instrument was constructed on the basis of the second level of the Information Processing Taxonomy (IPT) Model (Fong, 1992). According to the Model, at this level, some respondents' requirements are specified. They are : (a) information is acquired from external source and evoked from Long Term Memory (LTM), (b) type A information which is evoked from the LTM refers to the facts, skills, concepts, algorithms or principles of a mathematical topic on which the question is constructed and (c) the problem solver is required to link the data acquired from external sources with the invoked information from LTM to work out the solution. The following is a word problem used for the study:

> "Some stamps are shared between Weichi and Yihua in the ratio 7:3. Weichi has 184 stamps more than Yihua. All the stamps are then shared equally between them. How many stamps does each person get?"

Subjects and Sample

A sample of 499 pupils (aged 11+) were selected from seven schools of Singapore. Out of the total number selected, 249 were boys and 250 were girls. Using the School Mathematics Examination and Chelsea Ratio and Proportion Diagnostic (CRPD) test results as criteria for sampleing, 149, 219 and 131 pupils were categorised according to high, average and low ability respectively. (Note: The CRPD test results conducted in the Singapore sample showed a similar profile to the CSMS results except for a slightly better result at the upper end). The above question was administered to this sample of pupils.

Analysis and Results

Pupils' solutions were examined to determine their strategies applied in terms of their knowledge used and strategic paths of each strategy. The knowledge includes the concepts, skills, techniques or methods applied at certain stage of pupils' strategic paths in arriving at the solutions. Strategic paths refer to the sequence of knowledge presented in their solutions.

An analysis of pupils' solutions shows that all the strategies used to solve this problem could be classified under three categories: unitary method, proportion method and building-up method (Hart, 1984; Fong, 1992). When all the strategies are re-examined in depth in terms of the principles or skills (subsequently called skills) applied in the solutions, a form of pattern could be identified. It consists of a set of skills in sequential order listed as follows:

(a) The retrieval and representation of information,

(b) The computation of ratio units (whole or partial for each subject) from a given ratio,

(c) The sharing of parts/units between subjects,

(d) Writing equating statements,

(e) Applying the ratio and proportion concept to convert the ratio units to quantity (whole, partial or equal),

(f) Using the addition concept to find the whole quantity (number of stamps)

(g) 'Sharing method' to complete an equal quantity for each individual

The following paragraphs briefly describe each skill identified above :

Skill (a): Retrieving and Representing Information

Information encoded in the memory is found to be in one of the following forms :

(i) ratio of stamps = 7:3

(ii) Total : w : y : D

10 : 7 : 3 : 4

(iii) w/y = 7/3 = 7 : 3

Skill (b): The computation of ratio units (whole or partial for each subject) from a given ratio

Computation here involves finding the sum and difference between the ratio units of the given ratio 7 : 3. For example the total number of units = 7 + 3 = 10 units and the difference between units = 7 - 3 = 4 units.

Skill (c) : Sharing of Parts/Units Between Subjects

Two methods of sharing were observed : (a) sharing the whole units (i.e. 10 units) and sharing the difference of units between them (i.e. 4 units). For example :

(i) sharing the whole units.

Equal parts for each subject = 10/2 = 5

(ii) sharing the difference units.

The difference = 7 - 3 = 4

4/2 = 2 units

Parts to be shared = 2 units

Skill (d) : Writing Equating Statements

The equation '4 units = 184 stamps' is the main skill applied in every strategy to solve the problem. The statement 'Weichi has 184 stamps more than Yihua' and the earlier computed figure 7 - 3 = 4 units probably evoked the equating statement above.

Skill (e) : Applying the Ratio and Proportion Concept to Convert Ratio (Whole, Partial or Equal) Units to Quantity (number of stamps)

Converting ratio units to quantity is one of the most essential skills in solving the question. The following examples show the various methods to convert the whole units from the equating statement '4 units = 184 stamps'.

(i) unitary method: 4 units = 184 stamps

1 unit = 46 stamps

10 units = 460 stamps

(ii) proportion method : 4 units = 184 stamps

10 units = (184/4) x 10

= 460 stamps

(iii) building-up method : 4 units = 184 stamps

2 units = 92 stamps

10 units = 460 stamps

Skill (f) : Using the Addition Concept to find the Whole Quantity of Stamps

Two different procedures of using the addition concept were observed in the problem solving processes. They are summarized below :

(i) Addition concept to compute the whole quantity :

Adding the smaller quantity to the bigger quantity

= 322 + 138

= 460

(ii) Addition concept to compute individual and then the whole quantity :

Adding the smaller quantity to the difference in quantity

= 138 + 184

= 322

Adding the smaller quantity to the bigger quantity

= 138 + 322

= 460

Skill (g) : Sharing Methods to Compute Equal Quantity
Obtained for each Individual Subject

Three alternative sharing techniques were observed from pupils' solutions:(i) using 50% of the total quantity for sharing equally, (ii) division of the total quantity (stamps) by 2 and (iii) division of the total ratio units by 2. They are illustrated as follows :

(i) using 50% technique:

50% of 460 = 230

(ii) using division of quantity technique:

Total quantity = 460

Each person gets 460/2 = 230

(iii) using division of units technique:

10 units = 460

5 units = 460/2 = 230

Analysis of pupils' strategies and the skills (summarized above) applied in solving the problem shows that they can be generalized in the form of a network connecting all the essential skills. The following summarizes the paths of strategies showing the connections between skills in linear form:

a-b-d-e
a-c-b-d
a-d-b-e
d-b-e
a-d-e-g
a-d-b-e-g
a-d-e-f-g
d-e-f-g
a-d-e-g-f
a-d-c-e-e
a-d-e-f-f-g

Schematic Model for Analyzing Pupils' Errors in Solving a Ratio and Proportion Problem

While analyzing the problem solvers' strategies in solving the ratio and proportion problem (as described in the previous paragraphs), some errors were observed. For example, the computation errors in the four operations or fractions, reading and encoding information. The following are some specific errors:

Specific Error	Example
Division	(322+138)/2=285
Subtraction	7-3=5
Addition	7+3=11
Fraction	(184/4)x5=46
Encoding	7u=184
Information	(instead of 4u, the problem solver encoded 7u)

Some categorial errors seem to emerge when pupils' errors were further examined. They could be classified in terms of pupils' schematic knowledge. These categories are (1) no solution, (2) using irrelevant procedure, (3) incomplete schema with no errors, (4) incomplete schema with errors and (5) complete schema with errors. Basically, the principle behind the classification is whether there is any schematic error or specific error classified by the classical approach (Roberts, Engelhardt, etc.). The following paragraph illustrates examples showing the five different schematic errors:

1. No Solution

The first category, 'no solution', refers to a solution which has no written responses. In terms of schematic explanation, a problem solver is unable to connect or relate any of his available schemas to the information obtained from the question. As a result of this, no solution is presented.

2. Using Irrelevant Procedures

The name "using irrelevant procedure" is given to the second category of errors. For this category, the problem solver is unable to retrieve any relevant knowledge or information and, if any, apply it to work out the solution. Any knowledge or information which is retrieved has no connection or link to the question although the problem solver may assume that those pieces of information retrieved are the best possible solutions. The following are two examples taken from pupils' solutions in solving the question that illustrate this category of errors.

Applying *irrelevant procedures* :

(i) 184x7=1288
184x3=552

(ii) 182/2=92

In example (i) above, the problem solver merely retrieved the given information (184, 7 and 3) from the external source and a stored procedure (i.e. multiplication concept) from the LTM. The information, 184, 7 and 3, is connected or operated to produce the result as shown in (i) above. Notice that the procedure of multiplying 184 by 7 has no relevance to the requirement of the Question. For example, the figure 184 is not the value (number of stamps) representing one unit. Thus the multiplication procedure could not be justified. This step becomes irrelevant if the value 184 is multiplied by 7 or 3. Hence, the procedure is classified as "using or applying an irrelevant procedure". In terms of schematic explanation, it might be inferred that the problem solver has some form of schematic knowledge such as multiplication procedure. By connecting two figures and the multiplication concept, the result 7x184=1288 is produced. In one way or other, the problem solver has connected this procedure to the given information without realizing that the connection is erroneous.

3. Incomplete Schema with No Errors

The third category of error is named as "incomplete schema with no errors". For this type of error, only some correct steps of the strategy are presented by pupils in the solution. From the fact that some pupils are not able to complete the solution, one could possibly infer that the problem solver has limited or insufficient schema or he/she is unable to connect all the relevant information that leads to the solution. No actual error is made other than incomplete retrieval of schema leading to the solution. The following paragraphs illustrate an example which explains a complete schema obtained from the problem solvers' solution followed by some examples of incomplete schema identified from pupils' solutions.

A complete schematic solution for the Question:

	Skills
w : y = 7 : 3	<---------------- (a)
7 + 3 = 10	
7 - 3 = 4	<---------------- (b)
4u = 184 stamps	<---------------- (d)
1u = 46 stamps	
10u = 460 stamps	<---------------- (e)
(460/2) = 230	<--------------- (g)

The strategic path of this solution is a-b-d-e-g.

Notations : (a) Retrieval and representation of information
(b) Adding or subtracting two ratio units
(d) Writing equating statements
(e) Applying proportion concept to convert ratio unit to quantity of stamps
(g) Sharing method to compute each quantity of stamps for each individual subject.

Incomplete correct schemas identified from problem solvers:

(1) a-
(2) d-e-
(3) b-d-e-
(4) a-e-

Notice from the incomplete schemas above (2,3 and 4) that problem solvers have left out or were unable to use skill (g), the sharing concept, to compute the number of stamps each person gets. They could not proceed beyond categorial skill (e). The results shown here seem to imply that some problem solvers have insufficient schema or unable to connect the relevant schema to complete the solution. Specifically, there is a lack of sharing concept in the problem solver or he/she is unable to connect it to other concepts that lead to the incomplete schematic error.

4. Incomplete Schema But With Errors

The fourth category of error (which is related to the third category above) is named as "incomplete schema but with errors". Apart from demonstrating incomplete schema or unable to connect all relevant schema, problem solvers also make other types of specific errors such as encoding of information and subtraction of numbers (see examples in the following paragraphs). Using the complete schematic solution as shown above for comparison (strategic path a-b-d-e-g), the following incomplete schemas are detected. Those letters with brackets in the first column below (Erroneous Strategic Path) are steps which contain mistakes. The second column indicates the step and explains the actual mistakes made by the problem solvers.

Incomplete schemas with errors identified from problem solvers

Erroneous Strategic Path	Example
(1) a-(d)-	w:y = 7:3 <------------ (a) 7u=184 <--------- (d) (classified error : encoded a person's ratio unit in writing equating statements)
(2) (b)-d-e-	7-3 = 5 < ----------- (b) 5u = 184 <----------- (d) 1u = (184/5) 10u = 368 <----------- (e) (classified error : computation in subtraction)

In (1) above, the problem solver made an error by equating the ratio unit of Weichi (i.e. 7 units) to 184. Notice that the statement "Weichi has 184 stamps more than Yihua" might lead the problem solver to encode "Weichi has 184 stamps" and ignore the last part of the statement "more than Yihua". The error made here could be classified as "encoding wrong information".

In (2) above, a mistake is made at the very beginning of the solution when subtraction is carried out to find the difference between two ratio units. For steps d and e, the principles used are correct but follow-through errors are made.

5. *Complete Schema with Errors*

The last category of schematic error identified is "complete schema with error(s)". This type of schematic error arises when an error is made in computation or encoding of information although a problem solver is able to connect all relevant schemas to the problem's requirement. Table 2 shows some examples of complete schematic solutions but with some errors made in the solutions' paths. Those letters with brackets in the first column (Erroneous Strategic Path) are skills which contain errors.

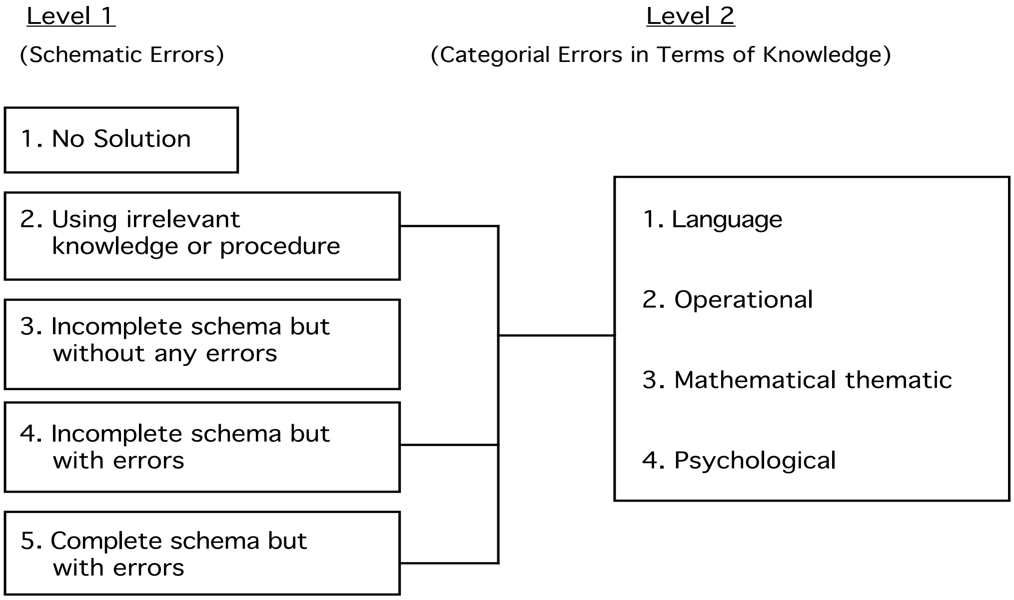
Table 2 : Examples Showing Complete Schematic Solutions with Errors

Erroneous Strategic Path	Example
(1) a-b-d-e-(f)-g	f:(322 + 138)/2 = 285 (classified error : computation in division)
(2) (b)-d-g-e	(b) : 7-3 =5 (classified error : computation in subtraction)
(3) a-d-e-(f)-g	(f): 322+138+184 = 644 (classified error : misconception of addition concept)

Discussion

The analysis of errors from the preceding paragraphs and review of the previous studies on error analysis lead the author to conceptualize two levels of errors in solving word problems in this study. The first level is categorized in terms of strategic schema. With respect to this, five categorial errors are identified: no solution, using irrelevant procedure, incomplete schema with no errors, incomplete schema with errors and complete schema but with errors. The second level can be categorized in terms of the classical ways of classifying errors. For example, the errors can be classified as computation, language and encoding information. From these two error classifications, a network of errors in two levels emerges and they are summarized to show their relationships in Figure 2.

Figure 2. A Network of Two Levels of Errors of Problem Solving



The Figure 2 shows a set of Level 2 type of errors which is sub-classified under categories 2, 4 or 5 of the Level 1 type of errors. For example, "complete schema but with errors" is a category of the Level 1 errors. Within this class of errors, a set of Level 2 type of errors such as language, operational, mathematical thematic and psychological knowledge is subsumed under it.

Examination of the various types of mathematical errors from the Level 2 shows that errors were classified on the basis of pupils' written solutions. They seem to lack analysis of children's cognitive processes in greater detail. For example, the language category of errors does not show the actual processes why pupils do not understand the problems administered to them. The mathematical themes show only the have or have-nots of certain themes. They have not gone into the investigation of the schematic aspect of learners' thinking. Although the operational knowledge has related to schematic theory, the various error types are sporadic. The present analysis of the different types of errors is meant to supplement the shortcomings of the previous studies. It looks into the problem from the schematic perspective. Besides reorganizing the various types of errors (See Figure 1), the first level in terms of schematic knowledge is also developed.

The present 2-levels model provides researcher with a further insight to look into pupils' errors from different perspective. Probably in solving problem, a pupil is first required to overcome the first level which means that he/she needs to acquire a complete schema to tackle a problem. At this level, he/she is required not only acquiring essential knowledge to solve the problem but also to organise them so that a solution is found. Thus it is important to look into pupils' errors from this perspective in order to help pupils' with mathematical difficulties.

The present model has implications to the teaching of mathematics and mathematics education research. For the former, teachers who are aware of the model in classroom teaching are able to point out the extent of children's schematic errors. The model also helps teachers to emphasize the critical points (schemas which the problem solver lacks) where children make errors most frequently in their teaching. Being aware of the model, teachers are able to identify pupil' status (ability to solve problems) by following a similar scheme when children's work is examined. For the latter, the analysis of different types of errors in schematic form provides a further insight of mathematics education research which involves the application of cognitive psychology in explaining children's errors in problem solving. However, introspection of the error types in problem solving and its relation to cognitive perspectives gives rise to a list of questions to be answered in future research. The following are some possible research questions for investigation:

(1) Is the hypothetical error analysis model above (a network of two classified levels of errors) comprehensive and applicable to all mathematical problems?

(2) Could the hypothetical error analysis model be improved and expanded to include other classified levels of errors?

(3) The strategic paths represented by cognitive processes of problem solvers have been explained by the IPT Model in the present study. To what extent does the Model could explain the erroneous schema of problem solvers?

(4) What are the implications of research in schematic error analysis to the teaching and learning of mathematics?

In carrying out the research, a list of mathematics topics at the primary level could be considered so that the model(s) on the error analysis can be verified using the whole range of mathematics topics.

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