Third Misconceptions Seminar Proceedings (1993)

Paper Title: **Metacognitive Monitoring of Mathematical Concepts Through Children's Writing in School Mathematics**

Author: Ellerton, Nerida F. & Clements, M. A. (Ken)

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Keywords: Educational methods,concept formation,,writing strategies,metacognition,thinking skills,mathematical concepts,learning processes,
General School Subject: mathematics
Specific School Subject:
Students: elementary and secondary

Macintosh File Name: Ellerton - Mathematics
Release Date: 5-27-1994 H, 11-8-1994 I

Publisher: Misconceptions Trust
Publisher Location: Ithaca, NY
Volume Name: The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics
Publication Year: 1993
Conference Date: August 1-4, 1993
Contact Information (correct as of 12-23-2010):
Web: www.mlrg.org
Email: info@mlrg.org

A Correct Reference Format: Author, Paper Title in The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Misconceptions Trust: Ithaca, NY (1993).

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Metacognitive Monitoring of Mathematical Concepts Through Children's Writing in School Mathematics

Nerida F. Ellerton
Edith Cowan University
Australia

M. A. (Ken) Clements
University of Newcastle
Australia

After drawing attention to the growth of interest in different types of classroom activities that involve the writing of mathematics, this paper focuses on the idea that, through writing mathematics, learners can be assisted to monitor their own mathematical thinking. It is argued that the metacognitive advantages deriving from writing in mathematics can be of assistance not only to learners of all ages, but also to teachers in the sense that an analysis of students' writing in mathematics can provide a window into the mind of the writer.

WRITING MATHEMATICS

It has been claimed that the current interest in writing to learn mathematics was sparked by the "writing across the curriculum movement" (Fulwiler, 1987; Menon, 1992) which, in turn, owed much to earlier established theoretical links between written language and learning (Smith, 1982; Vygotsky, 1962). Before this, many commentators on education believed that language and mathematics, in general, had little in common, and that writing and mathematics were unrelated.

However, during the past decade, research papers linking language and mathematics learning began to be published in many parts of the world (see, for example, Cocking & Mestre, 1988; Ellerton & Clements, 1991; Zepp, 1989), and in particular there has been much interest in the idea of writing to learn, and writing to teach, mathematics (see, for example, the reviews in Connolly & Vilardi, 1989; Ellerton & Clements, 1991; Menon, 1992).

In Australia, as in other countries, there is increasing interest among mathematics education researchers in children's writing in mathematics. The "writing in mathematics" movement has been a particular focus of Australian mathematics education researchers; at least five chapters, for example, of the recent book *Communicating Mathematics* (Stephens, Waywood, Clarke, & Izard, 1993) deal specifically with research into children writing mathematics. This paper is intended to bring out some of the unique aspects of the "writing in mathematics" movement in Australia. Elsewhere (Ellerton & Clements, 1991, Chapter 5) we have provided a review of the literature of the Australian approaches to "writing in mathematics," and have attempted to identify major research issues emanating from the movement (Ellerton & Clarkson, 1992; Ellerton & Clements, 1992).

TYPES OF "WRITING IN MATHEMATICS"

Before any realistic assessment of the effects of the "children writing mathematics" movement can be made, the different forms of writing encompassed by the term "writing in mathematics" need to be defined, and the aims of the various activities embodied in these forms outlined. Swinson (1991) identified a range of writing activities used in school mathematics programs and discussed the use of techniques such as impromptu writing prompts, letter writing, re-writing, journal writing, summarising activities, essays, and writing in mathematics term papers. He concluded that "there are many ways that writing can be used to foster a better understanding of mathematics and to highlight areas where students are experiencing difficulty." Swinson went on to say that "if used at an appropriate time with suitable preparation, writing is an excellent aid for the teaching of mathematics" (Swinson, 1991, p. 43).

It is useful to summarise some of the types of writing in which children engage in many mathematics classrooms around the world. Figure 1, which is taken from a book titled *Using Writing to Teach Mathematics* (Sterrett, undated, c. 1992), summarises the types of writings used by Timothy Sipka, who teaches at Alma College in Michigan:

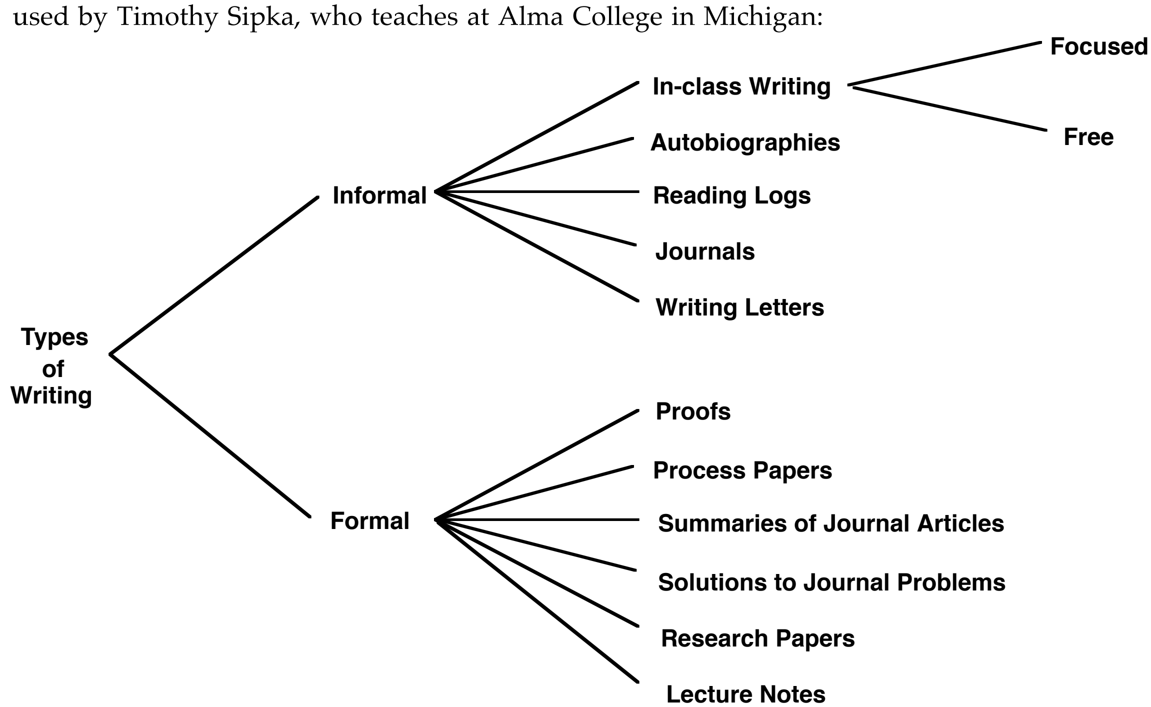


Figure 1. Types of writing in mathematics (adapted from Sipka, 1992, p. 11).

That mathematics educators are now taking the idea of writing mathematics seriously is evidenced from the fact that the book by Sterrett was published by the Mathematical Association of America. Readers interested in Sipka's (1992) distinctions between the various categories shown in Figure 1 are referred to his paper. In his paper, he also discusses how every category can be used in the classroom.

Later in this paper we shall draw attention to six forms of writing in school mathematics that are used regularly in some Australian mathematics classrooms, and in doing so will comment on the metacognitive benefits that might be expected to derive from each form. The six forms of writing are: (a) process/conference approaches; (b) problem posing; (c) journal writing; (d) interactive monitoring of students' learning of mathematics; (e) investigative mathematics project reports; and (f) projective essays or letters in which students respond to appropriate and interesting stimuli.

Australian elementary school teachers have a longer history of involvement than their secondary counterparts in the "writing in mathematics" movement. In fact, in the 1980s many elementary school teachers in Australia, filled with enthusiasm for modern language arts approaches, and especially those advocated by Graves (1983) and by Cambourne (1989), encouraged children to "write" mathematics on a regular basis. (see, for example, Bickmore-Brand, 1990, 1993; Ellerton & Clements, 1991; Waters & Montgomery, 1993). The adoption of what became known as "whole language approaches" in elementary mathematics classrooms was facilitated by funding from the Basic Learning in Primary Schools (BLIPS) program which enabled many teachers to participate actively in professional development programs such as "Exploring Mathematics In Classrooms" (EMIC), and Key Group (Ellerton & Clements, 1991; Robinson, 1986).

Secondary school teachers in Australia have also applied the notion of writing mathematics in their classrooms, and the results of this have been studied by mathematics education researchers. Waywood (1986, 1988), for example, conducted a longitudinal study on the effects of secondary students, at the school in which he was a teacher, making journal entries on a regular basis after mathematics classes. A particularly notable move in the direction of senior secondary students writing mathematics came when Victorian Certificate of Education (VCE) students were required to undertake Common Assessment Tasks (CATS) that involve extensive original research and associated creative writing (Ellerton & Clements, 1992; Stephens, 1993). As Mousley (1990) has shown, this had a profound effect on the roles of students and teachers. The requirement that students were to carry out the investigation virtually unaided, stripped teachers of their normal roles of setting learning objectives, transmitting the necessary knowledge and skills, and then assessing student performance. As students attempted to acquire sufficient background knowledge to enable them to put together written reports on nominated mathematical themes, school and local libraries were unable to meet the demand for specialist books. Assessment of these reports counted towards tertiary entrance scores.

EDUCATIONAL BENEFITS OF WRITING MATHEMATICS

The "writing in mathematics" movement has been linked theoretically with the constructivist movement in mathematics education (Menon, 1992), and there can be no doubt that, during the 1980s, the idea that mathematics teachers should provide learning environments that facilitated children's construction of their own mathematical meanings became a *cause célebre* among mathematics educators around the world. Children's idiosyncratic mathematical misconceptions came to be seen as potentially valuable expressions of the ways in which they were thinking. If the children's own versions of mathematics could be mapped, then the sequence and logic of their mathematical constructions would be better appreciated, thereby enabling teachers to provide learning environments which take greater account of the cognitive and affective needs of the children. In Piagetian terms, the teacher would have more opportunities for creating appropriate cognitive conflict situations; in Vygotskian terms, would be in a better position to provide appropriate scaffolding within the children's zones of proximal development.

Mathematics educators gradually came to realise, then, that teachers needed to find windows into their students' mathematical thinking, so that they, as teachers, would be in a better position to nurture the students' abilities to pose interesting questions and to develop appropriate problem-solving strategies. Thus, during the 1980s, researchers attempted to develop approaches which provided these windows. To this end, they studied "children's mathematics" from at least four different perspectives - mathematical problem solving, metacognition, constructivism, and language factors in mathematics learning.

As the idea of students writing mathematics became more widely adopted, teachers and researchers alike began to question the purpose of such an approach. In Australia, for example, the Mathematics Curriculum and Teaching Program (MCTP), which included a chapter on "Pupils writing about mathematics" in the first volume of its *Activity Bank* (Lovitt

& Clarke, 1988, pp. 239-258) listed the following seven points to support the idea that writing about mathematics enhances learning:

1. Mathematical experiences can be "captured" for later recall.
2. Pupils can use "natural" language arising from real contexts.
3. Writing, like talking, can facilitate internal organisation of mathematical relationships.
4. Pupils' work provides a springboard for discussion about the concepts being explored.
5. For many children, writing is an enjoyable, creative experience.
6. Writing can take place as a cooperative group task.
7. The written piece of work can assist teachers in formal and informal assessment.

Note that the use of the word "about" in the MCTP chapter title "Children writing about mathematics," raises the issue of whether from an educational perspective the distinction between "writing mathematics" and "writing about mathematics" is important.

However, not all mathematics educators were convinced that it was appropriate that ideas from language arts education should be translated so directly into mathematics education. McIntosh (1988), for example, argued that while Cambourne's notion of immersion in language made good sense in language arts settings since immersion in language is a fact of life for all children, it could well be inappropriate for mathematics education where immersion in mathematics has to be artificial and temporary, and manufactured by the teacher. And as Pengelly (1990) pointed out, language arts-inspired activities used in mathematics classrooms might provide more significant development in language than in mathematics.

A RESEARCH AGENDA FOR THE "WRITING IN MATHEMATICS" MOVEMENT

Until recently, mathematics educators had not defined an appropriate set of research questions and associated methodologies which together could inform the "writing in mathematics" movement. Research into the value of various forms of "writing in mathematics" tended to be carried out by individual researchers in different parts of the world, often in very idiosyncratic ways. This dearth of a holistic research agenda was partly attributable to the speed with which the "writing in mathematics" movement had grown, partly to the diversity of the forms of writing within the movement, and partly to the difficulty in deciding which, if any, of the results of parallel language arts research could be transferred to mathematics education settings.

Clearly then, a holistic research agenda was needed to give the "writing in mathematics" movement a firmer basis. Without such an agenda, the tendency to proceed in fairly random but attractive directions was likely to continue, which in the long term was likely to generate mixed results, culminating in disappointment and possible rejection by the majority of teachers of mathematics. In a recent paper (Ellerton & Clements, 1992) we attempted to define this agenda, drawing attention to five major research issues:

1. If students participate regularly in a particular form of "writing mathematics," are they likely to perform as well as students who do not participate in this form of "writing mathematics" on standard skills tests of mathematical skills, concepts and principles?
2. If students participate regularly in a particular form of "writing mathematics," are they likely to link more readily their mathematical understandings with their personal worlds?
3. If students participate regularly in a particular form of "writing mathematics," are they likely to become more efficient and effective at monitoring their own mathematical thinking so that they improve their problem-posing and problem-solving performances?
4. If students participate regularly in a particular form of "writing mathematics," are they likely to develop more positive affective responses to mathematics and mathematical situations? In particular, are they likely to develop feelings of ownership over the mathematics they construct or write about?
5. If students participate regularly in a particular form of "writing mathematics," are they likely to become aware of their own abilities, attitudes and preferences in mathematics, and be prepared to modify these in response to their own reflections?

Note that, with respect to investigative mathematics report writing, we (Ellerton & Clements, 1992) have formulated five additional research questions. These will be presented later in this paper in the section on investigative mathematics report writing.

METACOGNITIVE BENEFITS ASSOCIATED WITH DIFFERENT FORMS OF WRITING MATHEMATICS

An important emphasis in Australian research on "writing in mathematics" is the metacognitive benefits for children who regularly write mathematics. By "metacognitive" we mean the ability to monitor and reflect on one's own thinking processes - not so much on the content of what one is thinking about, but the ways in which one can become more conscious of one's own problem solving approaches (Mildren, 1992).

When students are prepared to express their mathematical thinking to paper, their ideas can become transparent. If students are able to provide some form of written account of their internal struggles as they attempt to solve mathematics problems, then teachers will become privy to students' metacognitive monitoring. That is to say, teachers will become more aware of the extent to which their students are in control of their own strategy development and are able consciously to attempt different methods.

1. Process/Conference Approaches

These approaches, largely inspired by Graves and by Cambourne, have been widely used in primary schools. Many articles and booklets have been written in which examples of children's "published" mathematics have been reproduced (see for example, Del Campo & Clements, 1987; Stephens & Caughey, 1986). However, the overall effects of these approaches have not been systematically evaluated. Reporting has often been confined to showing cute examples of children's writing, and we do not know whether children who do such writing on a regular basis achieve better understandings of mathematical concepts, are better able to link mathematical concepts with their personal worlds, and acquire healthier attitudes towards mathematics than do children who rarely, if ever, write any mathematics other than the narrow genre associated with pencil-and-paper exercises.

One of the dangers with the process writing approach in mathematics is that, when students "conference" with their teacher (that is to say, talk to the teacher about drafts they have written), the teacher is prone to superimpose her/his view of what constitutes an acceptable embodiment of a concept. From a metacognitive point of view, this can be confusing for the students, because by the very act of having written a draft, they are likely to have become aware of how they have sequenced their own thinking. If a teacher is too insistent on changing the draft, then students can lose confidence in their ability to control their own thinking. On the other hand, a sensitive teacher can use a conference as a means to find out how the children are thinking, and to assist them to become more conscious of the ways in which they have structured their thinking. If this occurs, then the process writing approach can assist the development of metacognition (Clements & Del Campo, 1990; McIntosh, 1988; Pengelly, 1990; Stephens & Caughey, 1986).

Research is needed, then, in which the educational benefits claimed for process writing in mathematics is carefully scrutinised. Some research in this area has been reported. For example, after reporting data on writing activities in some elementary and secondary mathematics classrooms in Australia, Marks and Mousley (1990) raised the issue of the purpose of the writing, and commented:

> If we are to set out to teach students to write different genres in mathematics, the role of the teacher is more than just to observe, prompt and react to students' efforts to express their mathematical ideas verbally, in writing or in other forms. It is also to teach the variety of genres valued in adult communication. (p. 132)

Marks and Mousley (1990, p. 133) went on to say that teachers should be actively teaching different mathematical genres as well as designing lessons with tasks that will require their use.

The Marks and Mousley article raises the question of who decides what mathematical genres exist - language arts experts, teachers of mathematics, mathematicians, or some combination of these groups. For example, it could be argued that the mathematicians' desire to achieve elegance in mathematical formulation and presentation, as well as their propensity to speculate in their writings on appropriate theories and models, and on what *might* be the case, all need to be taken into account in any definition of mathematical genres. Discussions between mathematicians and educators on such issues need to occur before decisions are made on appropriate genre.

2. Problem Posing

In the creation of a new mathematical problem, problem posers monitor their own thinking about how the elements and operations in the problem are going to relate, and what mathematical constraints apply and need to be taken into account in the problem formulation. For problems set in real-life contexts, the contexts are also likely to introduce constraints. Thus, metacognitive reflection is an essential component of the formulation of problems.

Yet mathematics curriculum developers and teachers have rarely asked students to create written mathematics problems on a regular basis. Certainly, from a constructivist perspective, it seems intuitively obvious that the challenge of posing mathematical problems would encourage learners to think about mathematical meanings and relationships, and therefore should empower them to construct mathematical understandings. But such intuitions need to be confirmed by research (Ellerton & Clements, 1992).

As Silver (1993) has stated, although “problem posing is an important aspect of mathematical activity and intellectual inquiry ... there is no coherent comprehensive account of problem posing as a part of mathematics curriculum and instruction, nor has there been systematic research on mathematical problem solving” (p. 86). Nonetheless, there has been a considerable body of Australian data reported in which children have posed their own mathematical problems (see for example, Ellerton, 1986a, b; Ellerton & Clements, 1991), and many mathematics educators have commented on the virtue of implementing a problem-posing curriculum in suitable problem-posing environments (see for example, Rice & Mousley, 1990).

Examples of two types of problem posing will be discussed in this paper. The first type was used by Ellerton (1986a, b) in her research, in which secondary school students were asked to make up (and solve) a mathematics problem "that would be difficult for a friend to solve." The task was open-ended in the sense that the students were free to choose any type of problem from any area of mathematics. Ellerton (1989) found that many students could not formulate meaningful problems, largely because they were unable to engage in the inner dialogue needed to ensure that the elements and operations were appropriately related to each other. Thus, for example, the Grade 10 boy who wrote the symbols shown in Figure 2 when asked to create a problem that would be difficult for a friend to solve, failed to realise that essentially the symbols he wrote did not define a meaningful problem. This boy had been studying the addition and subtraction of rational expression, logarithms and trigonometric functions, and apparently believed that any reasonably complicated combination of the symbols with which he had been dealing in class would define a difficult problem.

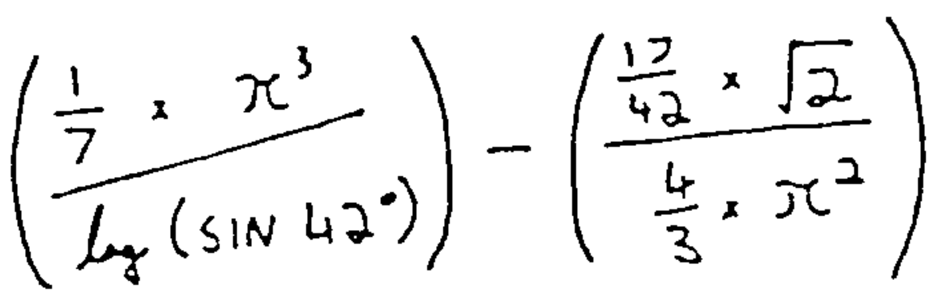


Figure 2. A meaningless "problem" posed by a Grade 10 boy (from Ellerton, 1989).

Ellerton's (1986a,b, 1989) research indicated that a large proportion of the 10,500 secondary school students involved in her study had a very narrow conception of what constituted a difficult mathematics problem. For most of the students, "difficult" equated to large numbers, many symbols and a lengthy combination of fairly simple operations. This perception of what constitutes a difficult problem seems to originate long before students enter secondary school.

The second form of problem posing which will be discussed in this paper is when students are asked to pose problems that are based on mathematical ideas that have been recently introduced and developed in the classroom. Del Campo and Clements (1987) described a problem posing classroom episode in which, following a demonstration and discussion associated with the relative strengths of cordial created from mixing cordial and water in different proportions. Students were asked to work in pairs and create and write down a related but different problem involving cordials of different strengths. The problem they created had to be such that they could solve it themselves. Once they had written down the problem and solved it themselves, they submitted both the problem and its solution to the teacher for checking, and if the teacher approved, then they were expected to present the problem to the whole class (materials such as cordial, water and clear glasses were available for this purpose).

Clearly, in both the types of problem posing referred to above, metacognitive thinking is required to create a good problem, and if, as in Figure 2, a "problem" is created which contains a fairly meaningless string of symbols, then this is evidence of the lack of metacognitive thinking. That is to say, the creator has not been able to link the mathematical ideas and relationships demanded by a sensible combination of the elements involved.

The within-a-context problem posing activity described above succeeded in motivating young students not only to think about the variables involved but also to express their ideas in writing. This type of problem posing is similar to that discussed by Brown and Walter (1990), who encourage students to examine problem conditions and constraints by asking questions such as "What-if?" and "What-if-not?"

3. Journal Writing

Research has suggested that educational benefits can derive from children writing, on more or less a daily basis, about what they believe they are learning or what they perceive to be happening during mathematics lessons (Mildren, Ellerton, & Stephens, 1990; Waywood, 1988). Students keep journals in which they write frankly, not only about what they are studying, but also about how they are studying, their difficulties, triumphs, frustrations, sudden inspirations, problem-solving strategies, plans for future action, and so on.

Waywood's school-based action research, conducted over a number of years at a secondary College in Melbourne, has demonstrated the potential of journal writing for helping learners develop "ownership" of what they do in school mathematics. Mildren et al. (1990) provided examples of upper primary school children monitoring their own mathematical thinking.

Waywood took steps to train students to do more than merely recall the mathematics they had done in class. He invited students to speculate on further extensions of and relationships between the mathematics they studied, and in this sense journal writing became more than a diagnostic or alternative assessment tool. His ideas were adopted by other mathematics teachers at his College, and an independent evaluation (see Clarke, Stephens & Waywood, 1989), revealed that all nine mathematics staff at the College were positive about the journals.

The students, for their part, reacted in a variety of ways to journal writing, with some students claiming that journals were a waste of time, and others stating that the need to make regular journal entries was useful because it forced them to explain to themselves what they were doing. One of Waywood's text categories, *dialogue,* is characterised by a conscious structuring of questions, providing evidence of a mind actively interrogating and attempting to make sense of the world. It appears that over the years some of the students learned how to write in a *dialogue* genre. Thus, for example, one student wrote reflectively of sin 60^{o}:

> The sin of 60 = 0.866025403... Firstly is the sin of 60 infinite I wonder? I think it is because you said the points on a circle is infinite. Then how could the square of 0.866025403... be exactly 0.75? If it is just an approximation then how could it equal exactly 1? Can you please explain? (p. 130)

There is much evidence of rich metacognitive thinking in this excerpt. Waywood (1988) commented that through the process of writing and receiving feedback students form a mathematical world view. For him, the dialectic of narrative, summary and dialogue reflects the dialectic of reporting on, describing, categorising and responding to the real world.

This research makes it clear that many people of all ages who are studying mathematics do not find it easy to reflect on the deeper meanings of what they are studying. It is often difficult to decide from journal entries whether a person has thought only at a shallow level or whether the person has reflected deeply but has not been able to express this on paper. Also, no answer has been provided to the question of whether regular journal writing improves scores on standard pencil-and-paper mathematics tests (such as those taken by students studying VCE mathematics).

4. Interactive Monitoring of Children's Learning of Mathematics

Clarke (1987) asked Grade 7 children in 15 Victorian secondary schools to provide written responses (confidential, but not anonymous) about once every two weeks throughout the year to a set of questions which asked them to report on (a) important things they had learnt in Maths during the past month, (b) types of problems they have found difficult, (c) what they needed more help with, (d) how they felt in Maths classes at the moment, (e) the biggest worry affecting their work in Maths at the moment, and (f) how Maths classes could be improved.

Clarke's large data set provided ample commentary on children's reactions to the overall impact of what was happening in their mathematics lessons and, in particular, whether they thought they were making satisfactory progress in the subject. According to Clarke (1987) most participating students believed that their involvement in what he called the IMPACT procedure had been personally beneficial, and over 80% of participating teachers likewise reported that their experience with the procedure had been valuable. While some teachers maintained that the IMPACT program had positively affected students' achievements and attitudes, some students complained that suggestions and criticisms they had made in their IMPACT responses had not been acted upon.

Unlike Waywood's efforts to train students to develop different mathematical reporting genres, Clarke's IMPACT procedure would appear to have had little effect on students' views on the nature of mathematical knowledge. Certainly, student feedback provided diagnostic information for teachers, but data obtained were more concerned with student'' "writing about mathematics," or "writing about mathematics learning," than with "writing mathematics."

From a metacognitive point of view, Clarke's IMPACT procedure asks students to reflect on how well they are doing at mathematics, and how they currently feel about their mathematics learning. While this may not appear to be capturing a high level of metacognitive thinking, it is non-threatening, and it could be argued that it provides a gentle introduction to children writing about their own mathematical thinking.

5. Investigative Mathematics Project Reports

Since the late 1980s in the state of Victoria (Australia), Grade 12 mathematics students wishing to qualify for the Victorian Certificate of Education (VCE) have had to complete a number of Common Assessment Tasks (CATS) (Stephens, 1993). The first CAT requires an extensive written report to be prepared by individual students on a series of reasonably open-ended questions requiring the development and application of mathematical principles to some focus defined by the Victorian Curriculum and Assessment Board (VCAB) in a special VCAB bulletin.

After several years of operating the new VCE mathematics, VCAB must surely have extensive sets of data and reports from the thousands of students who have submitted investigative projects. Something of the dramatic effect on pedagogy that followed the introduction of the projects was captured in the report by Mousley (1990).

Recently, the educational measurement issue of whether the assessment of the investigative reports can ever be sufficiently reliable for assigned grades to contribute fairly to tertiary entrance scores has received attention in the media. In the context of the present paper, however, we would draw attention to the need for research that answers questions such as the following:

1. What are the metacognitive benefits of asking students to write extended reports? Do students become more aware of the ways in which they think, and if so, does this awareness transfer to other mathematics problem situations?

2. When children write mathematics, are the expected genres of writing fundamentally different from the genres of writing that have characterised writing in school mathematics classrooms in the past?

3. Do students who satisfactorily complete an investigative mathematics report acquire research techniques that were never before required of secondary mathematics students?

4. Is the time spent in researching and writing an investigative mathematics report justified, given the criterion of satisfactory performance in subsequent mathematics courses?

5. Is the time spent in researching and writing an investigative mathematics report justified, given a criteria of being able to carry out independent research investigations in later life?

Many of the criticisms of the investigative reports have focused on the reliability of measurement issue, and on Question 4 above (concerned with preparation of students for tertiary mathematics courses). Such criticisms, it would seem to us, are narrowly based and fail to address the deeper and perhaps more important issue of whether the investigative reports have the potential to contribute to the development of a new and different school mathematics culture.

If the grades assigned to VCE investigative reports continue to contribute to tertiary entrance scores, then it is likely that school mathematics departments will decide to emphasise, to a much greater degree than at present, the preparation of written research reports. Ultimately this could generate school leavers who are more capable of designing, carrying out, and reporting research projects which require independent and sustained mathematical thinking.

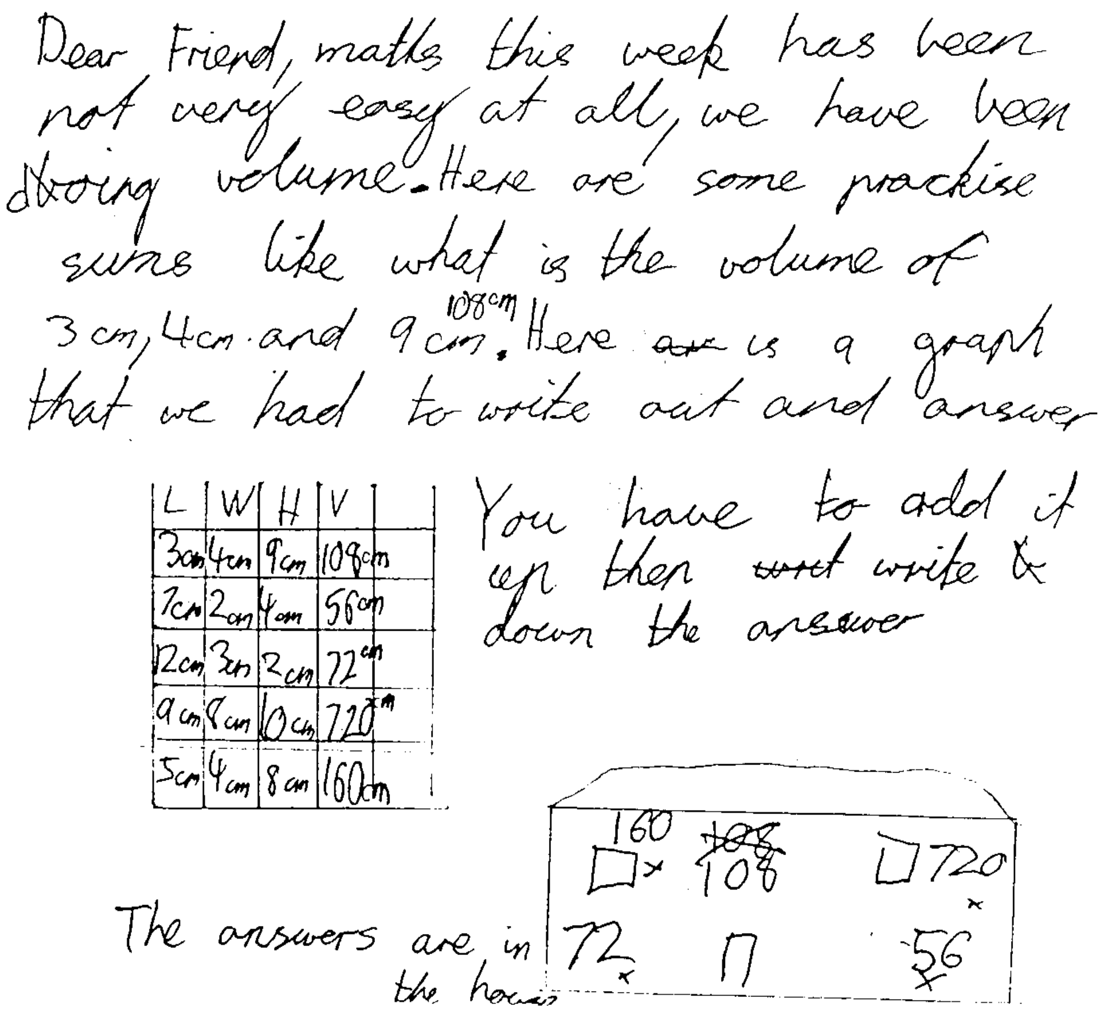
6. Projective Essays or Letters

Ellerton (1988) has investigated how children respond to the request to write a letter to a friend who has been ill. The following is the text of the request.

Imagine that you have a friend who has been ill and has missed about 3 weeks of school. Your friend has sent a message to you, asking you to explain what the class has been doing in mathematics so that he or she can do some extra work at home to catch up.

Write a letter to your friend describing what mathematics you've been doing in class. In the letter, don't forget to give examples of some of the mathematics questions that your friend would need to be able to answer so that he or she could catch up. Make sure you explain how to answer these questions. (p. 281)

The aim of this research was to obtain indirect projective data that would reflect what children think mathematics is, and what they perceive the purpose of school mathematics to be. Also, because the children were asked to create a problem and to say how to solve it, there was a level of metacognitive monitoring of their own thinking required as well.


Dear Friend, maths this week has been not very easy at all, we have been doing volume. Here are some practise sums like what is the volume of 3 cm, 4 cm and 9 cm. 108 cm Here ~~are~~ is a graph that we had to write out and answer

L	W	H	V
3cm	4cm	9cm	108cm
7cm	2cm	4cm	56cm
12cm	3cm	2cm	72cm
9cm	8cm	10cm	720cm
5cm	4cm	8cm	160cm

You have to add it up then ~~wort~~ write & down the answer

160 ~~108~~ 108 720
72 56

The answers are in the houses

Figure 3. Jay's letter to a friend.

Figure 3. Jay's letter to a friend.

The inescapable message is that, as a consequence of their school mathematics experiences, and of their unconscious acceptance of society's narrow views of the nature of mathematics, children become locked into a standard textbook/teacher/ examiner/parent view of mathematics. This is evident in the response by Jay (Grade 6) shown in Figure 3. Jay knows something about the calculation of volume, but apparently has not linked the formula $V = L \times W \times H$ with the fact that this applies only to cuboids. Also, his comment that "you have to add it up then write down the answer" seems to conflict with the multiplications he has carried out to produce the entries in the table. From a metacognitive point of view, Jay does not appear to have reflected in any depth about the notion of volume, or how this might be explained to someone else. He seems to have only a vague, somewhat mechanical knowledge of the idea of volume.

CONCLUDING COMMENTS

In this paper, a brief review of the "writing in mathematics" movement has been provided, with special emphasis being given to classroom practice and research in Australia. A research agenda for the "writing in mathematics" movement was also presented.

Particular attention was paid in the paper to six forms of "writing in mathematics" that are regularly used in some contemporary Australian mathematics classrooms. The six forms are (a) process/conference approaches, (b) problem posing, (c) journal writing, (d) interactive monitoring of children's learning of mathematics, (e) investigative mathematics project reports, and (f) projective essays or letters. With each of these forms of writing, reference was made to the likely metacognitive benefits for learners who engage in the writing. Also, the extent to which teachers would be likely to be made more aware, through what their students write, of the ways their students are monitoring their own thinking and responding to the classroom environments, was discussed.

There are other research questions which need to be addressed. For example, from the teacher's point of view, is the classroom time spent on "writing in mathematics" activities time wasted or is it time well spent from the point of view of developing students' mathematical understandings and positive attitudes? Also, from a cost-benefit point of view, do students' mathematics scripts provide teachers with sufficiently valuable diagnostic and predictive data to justify the time taken for students to produce the scripts, and for teachers to read and comment on them, and to maintain pertinent records? Also, certain ethical questions arise: for example, are teachers entitled to assess a student's journal entries? From a curriculum developer's point of view, does the new genre of mathematical writing demanded by extended projects (such as investigative projects) resemble the kind of writing in which professional mathematicians engage, and if so, is this a strong justification for modifying curricula and assessment?

Although these research issues are important, we have no doubt that great educational benefits are to be derived from the "writing in mathematics" movement.

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