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THE GEOMETRY OF CUBAÇÃO

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ABSTRACT

This paper is concerned with the description of the knowledge of 'cubação', a peculiar method of measuring land used by peasants in Brazil. A geometrical formulation of such a knowledge is given, and implications for mathematics and schooling are raised. Research in Science Education has very largely treated knowledge from an essentially individual point of view. In this work, however, knowledge is regarded as a social entity realized in individual discursive action. Knowing becomes being a participant in a discourse and to possess knowledge is turned into to be able to operate a certain kind of discursive process. One argument raised in the paper is that communal knowledge can be supposed to have a large tacit component; and, as such, it does have structuring rules which are not consciously available to those who are regarded as operating within them. Commonsense relates to knowledge at the level of this 'fundamental structure'. The attempt to formalize 'cubação' had then, to face the methodological problem of inferring tacit structures from interviewing data.

BACKGROUND

"Commonsense" is a broad notion used to indicate a cluster of ideas which in science education designate, basically, the ideas prior to teaching that students have, and which in some way contrast with those offered in science lessons. The persistence of these ideas despite instruction and their influence in children's learning of science have made a strong case for the many studies in the field in the last two decades.

It is possible to recognize in some studies, that the term commonsense has emerged as a qualification to ordinary/everyday explanations which are possible to describe using the modes of scientific explanations as some sort of control. One way in which researchers have attempted to give their studies greater generality is to focus on causality. For example, one finds causation being applied as (a) a broad category for designating "a common core for pupils explanations" (Andersson, 1986); (b) an ontological category for describing theory change in childhood (Carey, 1987); (c) a causal framework for analysing alternative conceptions (Gutierrez & Ogborn, 1992); and (d) an analytical principle for submitting curriculum materials to an analysis for the level of demand they make (Dal Pian, 1993).

One of the prevailing tendencies in the studies of commonsense in science education is to treat knowledge from an essentially individual, that is, psychological point of view, as a possession of or a property of people taken one at a time -whether in developement or learning, whether with respect to content ('concepts') or skills. The social dimension has been taken as context; as influential but as a set of factors to be held constant rather to be studied in its own right. Piaget's theory of cognitive development, Kelly's personnal construct theory and Osborne & Wittrock's generative learning model are some examples within such a tradition.

In recent years, however, the call for a more culturally oriented emphasis in the studies of commonsense became well documented: any elucidation of student's alternative concepts should require attention to the public conventions and social contexts of their proper use.

Researchers have sought to show how the sense of such concepts is connected to the ways in which they are used routinely in communicative situations. The various perspectives within which work is being done - phenomenological (e.g. Arcà, Guidoni & Mazzoli, 1983, 1984; diSessa, 1987; Hawkins & Pea, 1987); linguistic (e.g. Keil, 1981; Ogborn, 1985; Stavy & Wax, 1989); public informative (e.g. Ziman 1980, 1991; Lucas, 1983, Ogborn, 1987); ethnographic (e.g. Saxe, 1981; Hewson, 1986); social influential (e.g. Solomon, 1985, 1987, 1988; Lijnse *et al.*, 1990)- all attempt to reveal and account for knowledge changes (whether in cognitive or epistemological terms) that are part of ongoing life.

Within this approach, science is contrasted with commonsense, but researchers exhibit a considerable diversity in their concerns about the nature of the relationship. For example, while Solomon regards the social settings as a meaningful context, Ogborn considers theories of commonsense (understood as a kind of grammar which lies behind the use of commonsense explanations in ordinary discourse), and Lucas focuses on the different sources of scientific literacy (as a complex/set of conditions which would facilitate learning).

While development is usually assumed to occur, the conditions and mechanisms under which changes come about are hardly explained. Expressions such as 'cultural change', 'cognitive change', or 'bridging the cultures of everyday and scientific thinking' are relatively empty. If they have content, it is usually by analogy with Kuhn's idea of scientific revolution; that is, considering the relation between commonsense and science in terms of theory change.

A more recent and complementary approach to the 'Kuhnean' view is offered by Harré (1988) who emphasises the discursive character of explanations; that is, explanations, whether lay or scientific are discourses used by human beings to perform communicative acts. The approach brings out the idea of discursive community in relation to which the question of "why is this or that discourse explanatory" is discussed by Harré. It is this more socially oriented approach that we adopt. Within this perspective, knowledge can be understood and defined at the social level, when questions about it concern its means of social construction, reprodution and sustenance. Individual variation is now seen as context. In this sense 'knowledge' is not regarded as possessed by individuals, but rather as a social entity realised in individual discursive action. 'Knowing' becomes being a participant in a discourse and to possess knowledge is turned into to be able to operate a certain kind of discursive process. Taking this view, the possessing of knowledge (connaissance) is not a fundamentally characteristic of some individual. What the individual has is some set of competences in relation to that knowledge which are essentially discursive competences -the ability to join and to participate in a discourse. Thus, the criteria for somebody to be knowledgeable in something is the extent to which he is a functioning member of a discursive community in relation to which a field of knowledge (savoir) can be characterized. And this can have degrees. One can be a beginner, another can be more involved, another can be an expert. As soon as we accept this stance we are lead to ask questions about roles that people play in such a community.

There are two broad 'forms of knowledge' which have particular interest to us. One is science. The other is supposed to exist as knowledge supporting most human regularities in thought, feelings and behaviour, and which, in this sense, would be better called communal knowledge (not exactly commonsense). While the former exists and is formalized to be transmitted as an abstrach history, the latter is supposed to be found locally in human praxis. This does not mean that science exists detached from human praxis. Actually, what makes communal knowledge similar to science is the social perspective in which a kind of discursive practice can be seen in operation. Also, there is the possibility of describing them in relation to a discursive community.

The description of the knowledge of cubação which follows, is based on a portion of the data collected during a research study, intended to illustrate a case concerning communal knowledge, of relevance to science teaching. The study is reported in Dal Pian (1990).

Our theoretical starting point, discussed more fully in Dal Pian (1990), derives from an interest to clarify how communal knowledge can be conceived as a kind of 'representation'. Thus, we regard communal knowledge as public and being made by people. It is knowledge which exists as a discourse, and whose basis is social; that is, of the kind developed in the course of the division of labour and which refers to the particular activities involved. Communal knowledge refers to a culture and so can be seen as a system of representation with a proper stile, which distinguishes it from other kinds of representation like scientific theories or ways of reasoning in the past. Communal knowledge is then to be seen between alternative systems of representation; the understanding of each of these systems requiring, somehow, a re-understanding of the others' 'culture' in some imaginative projection of what it is about them that this particular one does not have.

General considerations about the nature of a formal structure for representing and describing communal knowledge led us to inquiry into the discursive character of communal knowledge. We distinguish two senses of the term 'discourse'. The first, which is not intended, is discourse meaning conversational or other exchanges. The second, which is that intended, is discourse in the structuralist sense, of a system which determines what it is possible to communicate. In the second sense, a discourse is characterised by general rules of formation and of appropriateness.

Communal knowledge is then seen as a regular system of knowledge whose structure is rule-governed. But if communal knowledge does have structuring rules, these are tacit and so, not consciously available to those who, nevertheless, are regarded as operating within them. It is argued that commonsense relates to knowledge at the level of this fundamental structure.

The attempt to formalize communal knowledge had, then, to face the methodological problem of inferring tacit structures from interviewing data. For the problem of dealing with deeper regularities presupposed by tacit reasoning, alternatives are available in science education in terms of the different approaches for looking at commonsense. In a previous work (Dal Pian, 1990) we argued in favour of a 'common-sense-referred view', in contraposition to a 'science-referred view'.

In the present paper, we will concentrate on the underlying 'geometry', formalized from interviewing farm-people and primary school teachers about cubação. Some implications for science teaching will be raised.

A LOOK AT CUBAÇÃO

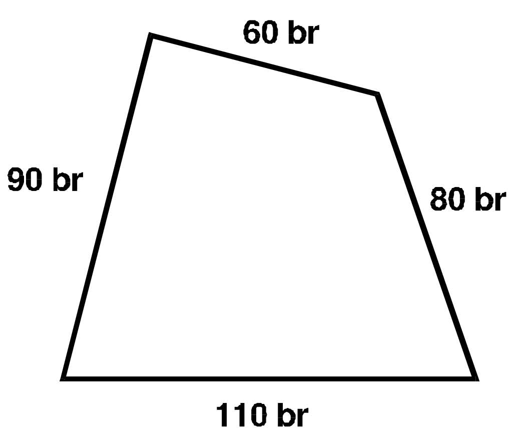
Cubação is a method used by farm-workers in the Northeast of Brazil to detemine the extent of plots of land. For more than a century it has been routinely performed as part of the work of agriculture, wherein the organization of fields for planting requires sets of plots to be delimited. The procedure is generally used by farm-workers in various situations in agriculture, including commercial transactions with farmers, agricultural technicians and inspectors of the Brazilian Bank. It is orally learned and orally transmitted from one generation to the other, through the work of agriculture; and possesses no relationship with the pedagogic discourse which goes on in schools.

In this Region, amount of land is usually expressed in numbers of *mil covas*, an old unit which has been known since colonial times to be approximately one third of a hectare, and to represent the amount of sugar cane transformed into sugar in a mill, in one day (in Portugese, *mil covas* would be translated as "a thousand pits on the ground"). Also, *mil covas* is known to be equivalent to a 'quadra' (square) of 625 square *braças*, one *braça* being approximately equal to 2.2 metres (Andrade, 1980).

So, if we take this 'quadra' as a unit of area, plots of land can be easily reckoned in *mil covas*: if lengths are measured in *braças* (br), one possible solution is to find the area of the plot in *square braças* and then to transform the result into *mil covas* (1 *mil covas* = 625 sq br); if lengths are given in meters, the result of the calculation can be obtained in hectares which is then converted to *mil covas* (1 ha = 3 *mil* and 305 *covas*). As relations between systems of units can be established, no question of dispute about results is expected to arise if farm-workers keep using the traditional unit: the hectare-system functions as normative.

However, to do this is not at all to perform cubação. The method is not recognized as such if a particular procedure is not pursued. This is a procedure in which measurements of lengths are carried out in the field in *braças*; and then, by means of a unusual succession of arithmetical operations, the 'area' of the plot is obtained in *mil covas*.

The sequence involves: **(a)** to add -two by two- the opposite sides of the tract, which in some way must be conceived of as a quadrangle; **(b)** to multiply the results of (a); **(c)** to multiply by 4; **(d)** to divide by 10; and finally **(e)** to check the results so obtained. For example, applied to the situation in Figure 1, cubação would give the area of the plot as 11 *mil* and 560 *covas* (or 3.5 hectares); and the following steps would have to be performed:



step 1: (90 + 80)
step 2: (60 + 110)
step 3: [(90 + 80) (60 + 110)]
step 4: (28900 x 4)
step 5: (115600 / 10)
step 6: 11560
step 7: checking sums

answer: 11 mil and 560 covas.

Figure 1

In other words, if we think of sides **a** = 90 br, **b** = 60 br, **c** = 80 br, and **d** = 110 br, the area of the plot is given by {[(a + c) (b + d)] 4/10}.

One striking fact about the procedure is that, whilst regarding *mil covas* as equivalent to an area in hectares distinguishes between "transforming units" and "calculating the area" (tacitly understood in terms of Euclidean procedures), cubação itself refers to a sequence of operations in which such a distinction is not required. It is also striking the idea that one same procedure can be used to estimate the area of all kinds of plots, of different shapes. Discrepancies soon arise when we consider the extent to which cubação gives an approximation to the "correct" area. Consider, for example, a trapezium as in Figure 2, and a 'square-circle' (Figure 3). The former is overestimated, and the second underestimated, by cubação (A_E = Euclidean area; A_C = area by cubação in *mil covas*; and $A_{C\to E}$ = area by cubação in Euclidean units).

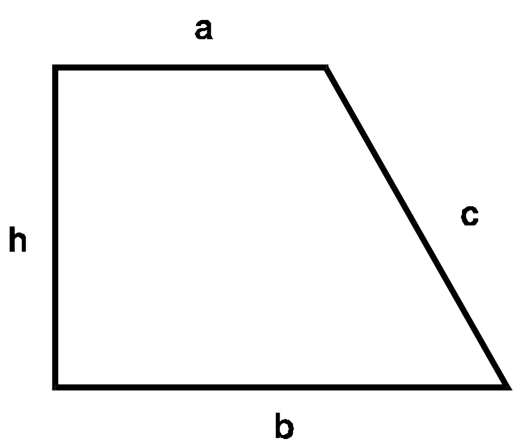


Figure 2

$A_E = [(a + b)/2]h$

$A_C = [(a + b)\ (h + c)]\ 4/10$

$A_{C\to E} = [(a + b)/2]\ [(h + c)/2]$

$A_{C\to E} > A_E$, as $[(h + c)/2] > h$

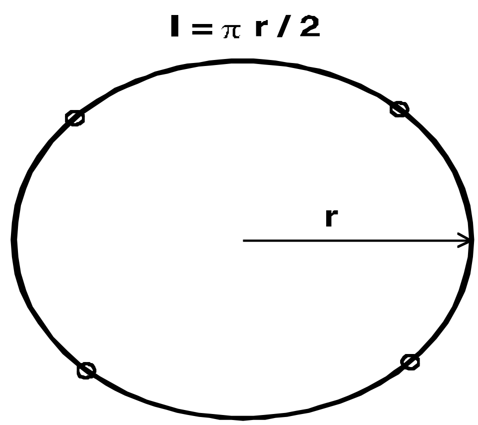


Figure 3

$A_E = \pi r^2$

$A_C = (\pi r)^2\ 4/10$

$A_{C\to E} = (\pi r^2)\ \pi/4$

$A_{C\to E} < A_E$, as $\pi < 4$

The fact that cubação has survived official attempts to introduce the hectare-system in the Region makes the situation more intriging. Having regard to the widespread and significant use of cubação in practices and negotiations in agriculture, the question arises of how valid it is as a method for estimating the area of land.

CUBAÇÃO FROM THE EUCLIDEAN PERSPECTIVE

To study the validity of cubação as a procedure for measuring area, it is necessary to 'transduce' the account of the procedure as given by farm-workers into an algebraic formulation capable of comparison with Euclidean procedures. A typical farm-worker's account would be:

"In the field, you have identified the position of your plot in terms of local features (rivers, roads, etc.) and its extent decided by identifying four edges, labeled West, East, North and South. You have measured each edge by means of a *braça*: suppose each is 25 *braças*. You have a '100-square'.

On the paper, write down the four '25s'. Add the 25-West and its opposite 25-East: 50; and the 25-North and its opposite 25-South: 50. Multiply the results: 2500. As you have four edges, multiply the latter result by 4: 10000. Ignore the last digit: 1000. Read the result as '1 *mil covas*'. You have 1 *mil covas*, the area of your '100-square'."

The re-expression would look as follows:

$$A_C = [(x_N + x_S)\ (x_W + x_E)]\ 4/10 \qquad \textbf{(1)}$$

with x_W, x_E, x_N, x_S in *braças* (br) and A_C in *covas*.

Applying this to the case of a square with side x, we obtain

$$A_C = [(2x)\ (2x)]\ 4/10 \quad , \qquad \textbf{(2)}$$

and the result as:

$$A_C = 1.6\ x^2 \ \ covas \quad . \qquad \textbf{(3)}$$

We can then conceive of the factor 1.6 as a conversion factor from br^2 to *covas*, and establish: 1 br^2 = 1.6 *covas*.

Thus in this case A_C can be normalized to Euclidean area units, and we can say that cubação gives a correct estimation of area for square plots. We can also show that cubação is correct, with the same normalising factor, for both *Regular Rectangles* (RR, as in Figure 4) and *Regular Angular Rectangles* (RAR, as in Figure 5).

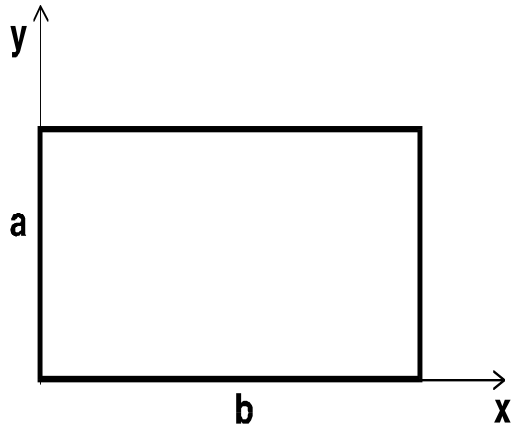


Figure 4

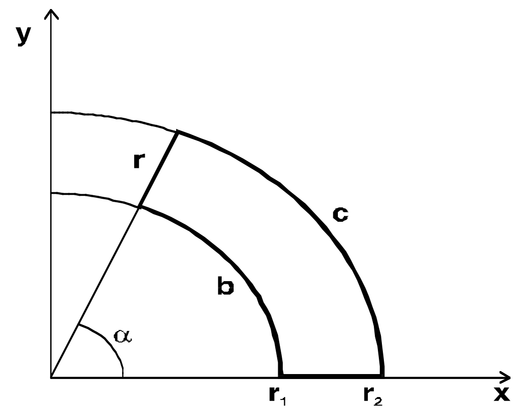


Figure 5

It is convenient to define a '*normalised cubação*' A_N by

$$A_N = [(x_W + x_E)\ (x_N + x_S)]/4$$

which preserves the same algebra as cubação but for which the area of a square is given directly in br^2. For the rectangle RR in Figure 4,

$$A_E = A_N = (2a\ 2b)/4 = ab \quad .$$

For the Regular Angular Rectangle RAR in Figure 5,

$$A_E = \int_0^{\alpha} d\theta \int_{r1}^{r2} r\ dr = (\alpha/2)\ (r_2^2 - r_1^2) \quad .$$

As:

$$r = r_2 - r_1 \quad ; \quad b = \alpha r_2 \quad ; \quad c = \alpha r_1 \quad ,$$

we can write:

$$A_E = r\ (c + b)/2 \quad . \qquad \textbf{(4)}$$

The normalized area by cubação (A_N) is:

$$A_N = [(2r)\ (c + b)]/4 \quad ,$$

which is exactly the same area A_E in (4). The fact of cubação being correct for an RAR, is perhaps unexpected.

Thus, cubação can be seen as an appropriate way of calculating the area of an RAR, of which the RR is a special case. The advantage of cubação lies in the fact that a wide range of different shapes can be reckoned by *the same and unique simple formula*.

It would seem at first sight as if cubação is particularly badly adapted to circular shapes. However, if for the *Angular Triangle* shown in Figure 6, we reckon its area by normalised cubação, we obtain

$$A_N = (2r\ a)/4 = (r\ a)/2 \quad ,$$

counting the 'fourth side' as empty.

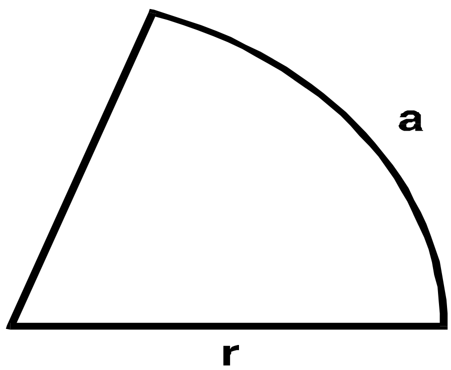


Figure 6

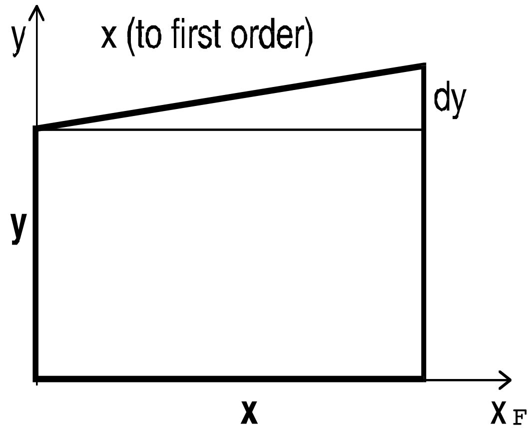
When $a = (\pi\ r)/2$, this is of course exactly the Euclidean area

$$A_E = (\pi\ r^2)/4 \quad ,$$

and by extension normalised cubação correctly gives the area of a circle of radius r, circunference C, as

$$A_N = (r\ C)/2 \quad .$$

It is easy to show that, to first order of small quantities, normalised cubação also gives the correct area of what we could call '*Irregular Rectangles*' (IR, as in Figure 7) and '*Irregular Angular Rectangles*' (IAR, as in Figure 8). That is, cubação is insensitive to small dilatations of sides, which contributes to the production of more accurate results in the case of reckoning general irregular regions.



igure 7

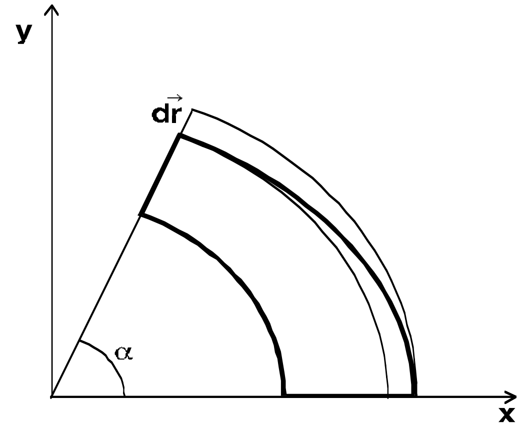


Figure 8

THE NOTION OF AREA IN CUBAÇÃO

Those of us who have been socialized into the mathematical Euclidean way of doing calculations of area have to hold as evident two things: one is a notion of what an *area* is; another is a notion that there is a *method* which is underwritten by being apparently able to be demonstrated to be the way to reach the area-idea you started with. Euclidean area and Euclidean method are tied together by a supposedly logical structure; and we have got to penetrate the organic whole of the issue before it makes any sense. We can not understand Euclidean area without knowing the method and we can not understand the method without the concept of area. And both can not be completely understood if we do not keep in view that what purports to hold them together is a logic. 'Area' belongs to an integrated theory. Thus, as soon as one introduces another method for estimating area, this theoretical circle is interrupted or suspended. If the farm-workers were thinking of area in the Euclidean sense, then cubação would look a strange way to work out such a thing. But farm-workers are thinking like a person planting things or buying land and they do not question the method.

When we try to make sense of the procedure by questioning farm-workers about different geometrical shapes for which the method fails, explanations are confidently put forward as if belonging to systematic knowledge (Dal Pian, 1990). Examples of the direct accounts given by the informants include:

. "it is not necessary to have a quadrangle with equal sides";
. "west must always be added to east, and north to south";
. "this tract has 200 *braças* (to designate the area of the tract);"
. "one multiplies by 4 because there are four sides";
. "one multiplies by 4 to 'see' the number of *mil covas*";
. "one should ignore the last digit to show the result in *mil covas*";
. "the expression *mil covas* does not designate the number of "*covas*" (plants or pits)".

In addition, results of interviewing experts in cubação and primary schools teachers suggest that some fundamental propositions could be taken as proper to the way of thinking about area in cubação, such as:

. "Any shape can exist. Some are known to people by experience. Others are not. In existing, these shapes have to have a given number of *mil covas* attached to them. This is so, even if one has never seen those shapes."

. "To each edge of a polygon, one can attach (make correspond) a fraction of the total area embraced by the perimeter."

. "Edges can be compensated without prejudice to the total area.

. "Cubação lacks a kind of parameter which is characteristic of the shape, without which it is not possible to accept that one unique procedure can apply to any shape."

. "There must exist 'a procedure of cubação' for the metric system whose description does not involve changes of units between systems."

. "It should be possible to conceive of a general procedure which has as a particular instance the case of cubação applied to a quadrangle."

As suggested by farm-people, there is a geometrical shape which seems to be beyond the scope of cubação in any imaginable sense: the circle. Circular shapes seem beyond the scope, not of cubação as a method, but of the farm-people's expertise. The circle is not simply recognized as a non-prescribed shape by cubação, but it is recognized to be beyond the competence of experts in measuring land.

These and other results from the empirical study have suggested that, for farm-people, the perimeter of an actual plot is itself a criterion as to its area. At the bottom of the farm-people's reasoning is the idea that shapes having the same perimeter have the same area.

Comparison of the areas of figures having equal bounding perimeters has been the focus of some old isoperimetric problems (see Heath, 1956); from which we learn, for example, that the conversion of any irregular quadrilateral into a regular one of equal perimeter is necessarily accompanied by an increase in area. Also, for regular polygons, it is known that the area of a circle is greater than that of any polygon with the same perimeter. In addition, the idea that the perimeter of a shape is itself a criterion as to its area has been considered by mathematicians as a misconception among non-mathematicians (see, for example, Heath 1956, pp. 332-333).

The idea of the impossibility of having different isoperimetric shapes with the same area is so strong in us, that it seems that the only pertinent question to follow when we look at the farm-people's performance in reckoning by cubação is "how inaccurate are the results?" But, given the properties of cubação just demostrated, it seems better to change the question to something like: "In what way is the perimeter of geometrical shapes a criterion for their area?" Thus we are led to ask about the general form of the shape for which cubação correctly indicates the area.

Let us define area A_P estimated by the perimeter, in the form

$$A_P = k\ P^2 \quad ,$$

where k, numerically equal to the area of a figure of unit perimeter, is characteristic of the shape of the figure.

In the case of a square, of side 25 br, perimeter P = 100 br, we have

$$A_C = [(25 + 25)\ (25 + 25)]\ 4/10 = 1000\ \textit{covas} \quad ,$$

$$A_P = k\ 100^2 \quad ;$$

where the parameter $k = 0.1$ *covas*/br^2. Thus, the procedure "ignore the last digit" in cubação (divide by 10 above) can be understood as the application of the constant k (the shape parameter) to an estimation of area from the perimeter. P^2 can be seen as a calculation of the "number of shape parameters" in the area, to be multiplied by the size of the shape parameter to obtain the area, or, as the farm-worker would say, "to 'see' the number of *covas*".

In other words, the perimeter alone does not determine the shape of the figure, which means that, in order to "see" an area, one needs, somehow, to "fix" the perimeter around a given shape. This is exactly the role of the shape parameter in the above formula. It gives *form* to the cells which compose the area and to the area itself. This geometrical fact leads to three important implications.

(a) First implication.

Any figure whose shape parameter is known can be estimated by cubaço simply by multiplying its value by the perimeter squared.

In this way, the perimeter constitutes area by way of the shape parameter which imposes 'rigidity' on the shape of the figure. Thus, for the quadrangle in Figure 9,

$$k_C = 1.6\ (\sqrt{3}/32) \approx 0.0866 \text{ covas/br}^2 ,$$

and the area A_C=866 *covas* (and not 1000 *covas*, as the case of a square of side 25br).

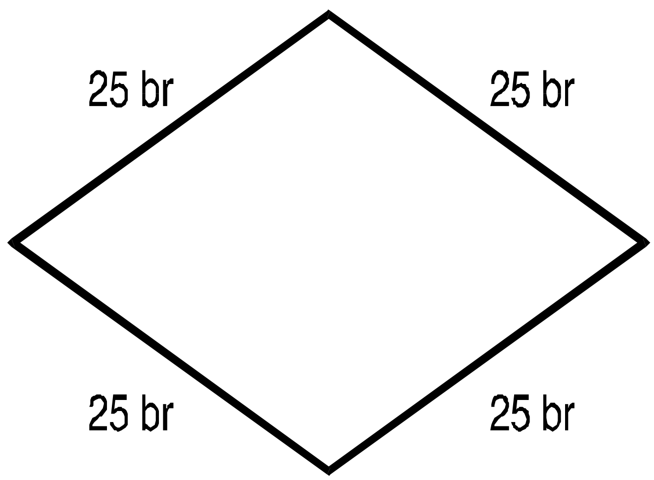


Figure 9

(b) Second implication.

Cubaço, as practised by farm-workers, consistently estimates areas A_C of shapes for which k_C = 0.1 covas/br^2.

We have shown that this is the case for RAR's and their special cases, rectangles and squares. By extension, appropriately interpreted, cubação also correctly estimates the area of circles, though now not following directly the procedure given by farm-workers. A class of 'squares' (k_C = 0.1) can be defined, examples of which are shown in Figure 10.

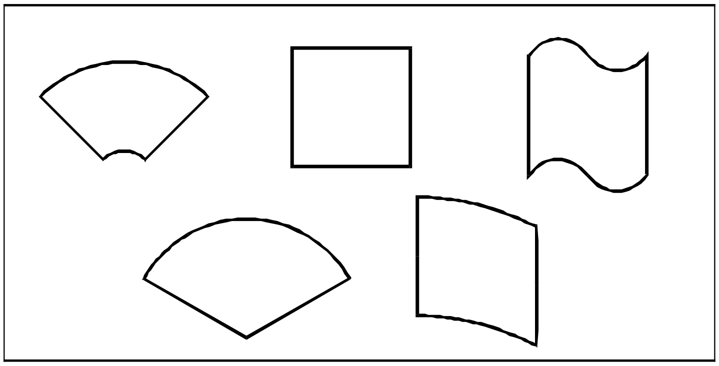


Figure 10

(c) Third implication.

Cubaço correctly gives the area of any shape constructed by the ends of a straight segment moved perpendicular to its present direction and/or rotated.

Some situations to which this proposition applies are represented in Figure 11. For example, shape (a) is obtained by the addition of a square ($A_{sq} = r^2$), a quadrant ($A_q = (r\ c)/2$), and a rectangle ($A_{rect} = r\ b$). The total area (A_T) is:

$$A_T = r^2 + (r\ c)/2 + (r\ b) \quad ,$$

$$A_T = r^2 + r\ (b + c/2) \quad . \qquad \textbf{(5)}$$

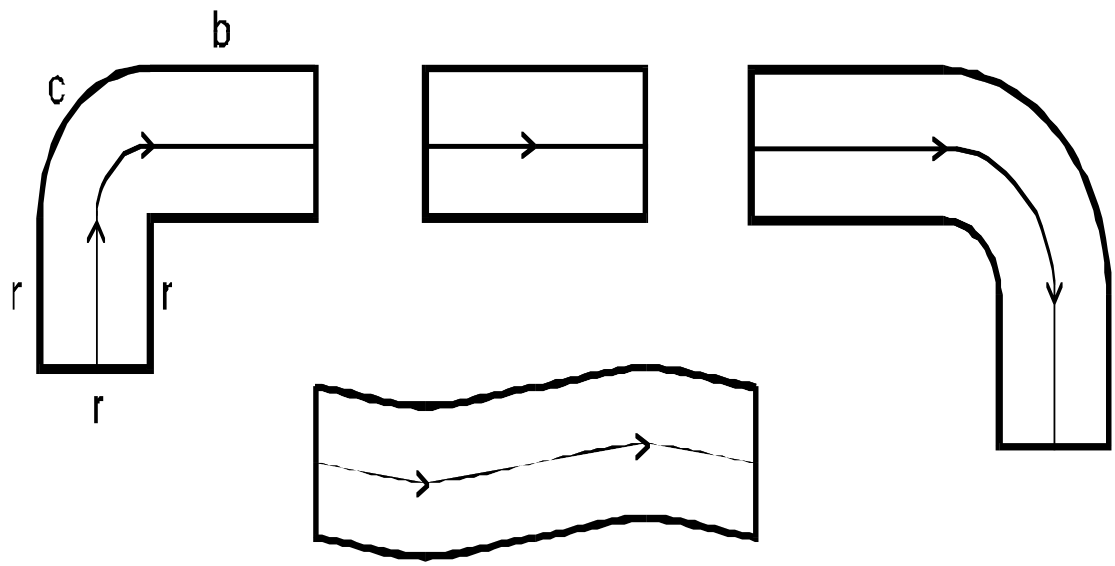


Figure 11

As a quadrangle with pairs of opposite sides given by [r & r] and [(r + b) & (r + c + b)], the total area of shape (a) is obtained:

$$A_T = [(r + r)/2]\ [(2r + 2b + c)/2] = r^2 + r\ (b + c/2),$$

which is exactly the same area A_T in (5).

IMPLICATIONS FOR SCHOOLING

From the preceding ideas, it follows that any plot marked out by ploughing successive strips will also be correctly estimated. The question arises of how much the traditional definitions of units of areas in terms of practical attributes try, exactly, to guarantee a correct application of the generic procedure presupposed by the system of measurement.

Consider for example the definitions of some units of area used in surveying in different places and times.

Acre. One *acre* is defined as the area of a 4-pole-strip which is 1 *furlong* long. *Furlong* comes from "*furrowlong*" and is generally suggested to be "the distance oxen could pull a plough before having to pause for breath".

Iugerum. One *iugierum* is defined as a rectangle of (120 x 240) *Roman feet* or (1 x 2) *actus*. The *actus* (120 *Roman feet*) constituted the basic unit of length in the Roman period, and literally meant "a driving", "the distance which oxen pulling a plough were driven before turning".

Feddan. The name *feddan* is applied (in Palestine) both to a unit of livestock in the *fellah's* farm and to the area which can be worked by that unit in a fixed period of time. In ordinary usage, a *feddan* means "a piece of ground which can be tilled, i.e. ploughed and sown, with a yoke of oxen in the space of one day".

The question makes sense when we realize that knowledge of surveying is characteristically algorithmic: it involves a set of procedures for isolating problems and solving them, a set of assumptions and permissible deductions, a way of thinking about things in which what is 'correct' about results is taken for granted rather than explicitly demonstrated. When procedures such as these are transmitted, they act as a check upon the body of transmitted facts, allowing them to be re-derived or excluded if no proof can be found (Gray, 1979).

Since cubação can be formulated as a surveying method, it becomes interesting to compare it with other systems of measuring land. The result will show that the underlying 'rationale' of the method turns out to be the same as the one which has existed in different places and times in history, such as the 'acre-system', the 'Roman-system' and the 'Aztec-system' of surveying. In so far as geometry is generally supposed to have its origin in measuring land, the question arises of what can we learn about more fundamental structures of reasoning about *area*, by both looking at actual/historical surveying systems and looking at the history of mathematics. From the former, it appears that methods of surveying are generally formulated in terms of the procedure (a, b, d and d = sides of a quadrangle):

$$A = \Phi \{[(a + b)/2] [(c + d)/2]\} \quad .$$

An account of the rule $\Gamma = [(a + b)/2] [(c + d)/2])$ expressed within the brackets { }, for the acre-system, can be found in Usill (1898); also in McEntyre (1978).

From the latter, we find that the rule Γ played an important role in the development of mathematics in nearly all ancient civilizations, remaining always attached to the procedure of finding the area of a quadrilateral as the average of one pair of opposite sides times the average of the others (Boyer, 1968, p. 18, p. 42, pp. 232-233, pp. 241-

243; Eves, 1969, p. 40; Dilke, 1971, pp. 15-17, p. 30; Seidenberg, 1973, pp. 180-181; van der Waerden, 1983, p. 207; Lauand, 1986, pp. 102-105). Particularly, with respect to the problem of computing Γ for a circle $[\Gamma=(1/4\pi)C^2]$, historians would suggest that expression $A_C = (rC)/2$ seems to have come first. Also, there is a suggestion that A_C refers to a relation which stands close to intuition/experience.

The relevance of such an analysis to school geometry relates to the fact that, underlying the methods of solving problems of area by Euclidean methods, there is the metric-system of units which is usually take-for-granted, and in this sense, 'ignored'. When one says, for example, that the area of a rectangle is given by the product of its sides, it is concomitantly presupposed that the result is given is square units of lengths, the same unit of length being used to measure the sides of a rectangle. It is also immediately supposed that the unit of area is a square having a side of one unit of length. And nobody asks why we measure lengths and areas in schools; or raises questions about the appropriateness and correctness of Euclidean procedures. Actually, it is not usual to look at the teaching of geometry, algebra or arithmetic as *practices* belonging to a *pedagogic discourse*.

If cubação can be understood as related to deep mathematical ideas, with a long history, related to the origins of the concept of area, this makes a difference to cubação as a *didactic entity*. Curriculum development usually *constructs* entities for transposing from science or mathematics that which can be understood as teaching about models and about the nature of space and objects. So, if we ask "why is a geometry course obsessed with area in terms of units of area, but not mathematicians", the reason could be addressed in terms of the well formed didactic object that the area represents. To accept that the concept of area is a construction has, then, some implications. It offers a different kind of potential for its educational interest and for the applications that one could find for it. It offers a possible relation of methods of measuring "here, now in Brazil", with the origins of the whole idea of measuring area. It helps us not to think in terms of "there is knowledge which we efficiently pass across"; but that "knowledge has structures of its own which are in the curriculum for didactic reasons."

By trying to understand cubação -and not just by saying how it is done- history itself can be made problematic. At the same time, by making history problematic, the question of cubação as a didactic construction can be transformed and potentially sets surveying methods in a historical mode of reasoning about area. Then other questions arise. For example, "is the nature of the content actually appropriate for the primary level of schooling?" It may even not be. It could be that cubação is best adapted to the history of mathematics. Or: "what is proper to teach about cubação at a given level of the primary school?"

FINAL REMARKS

School geometry knows nothing of cubação and we hope to have shown that it has interesting and rather abstract properties. It easily solves certain problems which would be difficult for the Euclidean geometer equipped only with squares to tessellate the plane.

The discursive character of communal knowledge -in the structuralist sense- was highlighted. To regard people's knowledge as belonging to a discourse does not mean that practical discourse can be always characterized as a discursive practice. This raises the question of how much more like cubação is waiting to be found. Cubação was, in a sense, 'discovered'. The knowledge of cubação demanded a construction to be built from more fundamental features of the 'perceptible experience' of farm-people (which is fully described in Dal Pian, 1990). This point reinforces the idea that any attempt to getting at new discoveries must contain an intention to grasp on how people's experience affect the process of understanding events, facts and entities in the world.

For schooling, the existence of communal knowledge not known to school implies that a confrontation has to take place. The paper has insisted on the necessity of getting at the historical level of analysis. To that extent, there are distinct discourses that can be seen about area and it is relevant to contrast them in their similarities and differences, if the attempt is to find a way of understanding the potential of cubação in teaching in the primary school. Particularly for science education in the Northeast of Brazil, it affords an insight into questions about *possibilities* for peasant students of learning another system of measurement and of relating this to cubação. More fundamentally, it affords an insight into the *reasons* to relate a new system to cubação and of the *value* of any other system of measurement to them.

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