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Seeing is Believing?
Students-Functions-Graphing Calculators

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The ability of a student to visualize functions is an important aspect of a mathematics education. Graphing calculators can quickly display the graphs of functions and evaluate function values. Such efficiency can contribute to explorations of families of functions. Therefore, graphing calculators have a major role, albeit not yet clearly defined or fully assessed, in the mathematics classroom today. Mathematics educators are faced with instructional technology issues as never before. An intrinsic concern is how, or if, the use of graphing calculator contributes to student understanding of mathematics content.

This paper describes some results of assessing the use of graphing calculators in selected sections of college algebra and calculus in a two-year college for three consecutive quarters. The rationale for requiring these hand-held computers is outlined. Calculator assignments and student writing assignments are described. The data from paired student-teacher interviews conducted outside of class is reported.

BACKGROUND

For a number of years, voices have been raised in support of educational calculator use. The invention of the hand-held calculator by Texas Instruments in the 1960's quickly caught the attention of mathematics educators (Harvey, 1992). This four-function calculator was soon joined by the scientific calculator. In 1980, the National Council of Teachers of Mathematics recommended that "mathematics programs take full advantage of the power of calculators and computers at all grade levels" (p. 1). Frankly, the national response was underwhelming; there was resistance on the part of teachers to the use of any calculators in classroom settings. Boyd and Carson (1989) reported on the failure of college developmental mathematics programs to employ calculator use.

The next great development in educational calculator use came with the introduction of graphing calculators by Hewlett-Packard. A graphing calculator is a hand-held computer which is programmable and makes graphics displays. In fact, a graphing calculator is also a scientific calculator. It is not appropriate to categorize graphing calculators simply as function graphers because the TI-85 displays vertical lines as well. The expensive Hewlett-Packard models soon had competition in the mid-1980's from much lower-priced Casio models. Keeping track of technological advances since then, with their associated impact on the teaching of mathematics, is challenging. As Kaput (1992) wrote,

> Anyone who presumes to describe the roles of technology in mathematics education faces challenges akin to describing a newly active volcano - the mathematical mountain is changing before our eyes, with myriad forces operating on it and within it simultaneously. (p. 515)

Nevertheless, whether anyone could keep up with the changes or was ready to implement them, graphing calculators began walking in the doors of college classrooms.

The National Council of Teachers of Mathematics in 1989 again presented recommendations on calculator use. This time the recommendation for grades 9-12 was, "Scientific calculators with graphing capabilities will be available to all students at all times" (p. 124). If this policy were decreed for high school students, then, by extension, it should apply to college students. The Mathematical Association of America (1991) stated, "Through the regular use of calculators and computers in collegiate mathematics courses, students learn more mathematics and can more rapidly apply that understanding in problem solving" (p. 6). Kaput (1992) stated that "computers are too difficult for the average teacher to use in the typical classroom on a sustained basis" (p. 517). Since graphics calculators are portable computers at a fraction of the cost, there are major advantages for the implementation of graphics calculators. The availability of moderately-priced graphing calculators may yet put a computer in the hands of each student.

INSTRUCTIONAL TECHNOLOGY

Teacher Training

Experience shows that the success or failure of new classroom strategies is often dependent upon the teacher. Kaput (1992) stated that at present it is unknown to what extent support is required to introduce teachers to new technologies. He wrote, "While technology provoked innovation and in a sense makes it possible, the important changes are those that follow from changes in the teacher's beliefs about mathematics, teaching, learning, students, and the appropriate use of classroom time" (p. 549). The question may well be how much teachers believe in the efficacy of classroom technology and how much they are interested in keeping up with rapidly changing technology tools.

Given the proliferation of calculator models described earlier, it is intimidating to some teachers to be required to provide instruction on several different machines. It may be beneficial to a teacher to initially restrict student choice to one model. Such policies should be listed in the schedule of classes so that students will be forewarned of expectations in this regard. Administrative approval for such requirements may also be an obstacle.

Some teachers have stated that they need to have the use of technology-augmented textbooks. Other teachers wish to have a curriculum guide prepared for their use. There are at present few textbooks which incorporate graphing calculators, although this situation is rapidly changing. When a textbook incorporates graphics technology, it runs the risk of either being generic and of limited usefulness to the novice or is brand specific and outdated due to the rapid pace of technological advances.

Student Training

There is a dearth of material regarding helping students learn to use graphics calculators. Teacher technology training is no model for student training given that teachers bring to the learning environment a mathematics background that is more conducive to the training objectives. Experience shows that it is problematic to assume that students will readily assimilate keystroke instruction. The statement, "Harvey and Waits reported that older students seem to have few difficulties changing the scale on a viewing window so a complete graph can be seen" (Senk, 1992, p. 133) needs clarification. Counterexamples to this claim can readily be cited.

Thompson (1992) noted that there has been little research on "the extent to which teachers' and students' conceptions interact during instruction" (p. 141). It is not clear whether the differences in teachers' and students' conceptions are diminished or exaggerated as a result of incorporating technology in the classroom. In the same light, there may be evidence to suggest that the technology assists the students who have rich mathematical backgrounds while the weaker student finds the technology another barrier to success.

Assessment

There are many unknowns in the appraisal of calculator use in mathematics. Kaput (1992) asked, "How do different technologies affect the relation between procedural and conceptual knowledge" (p. 549), and "How do social patterns change in mathematics classrooms that are technologically rich?" (p. 550). McLeod (1992) stated, "It seems likely that technology can play an important role in changing beliefs about mathematics and possibly even in improving attitudes toward mathematics" (p. 588). For meaningful incorporation of the graphics calculator into mathematics curricula, progress must be made in changing beliefs about mathematics and in understanding how one learns mathematics.

Educators and researchers are faced with the task of investigating how calculators can meaningfully support the goal of concept formation. Kaput (1992) asked, "The question to be answered is, `Will the technology help us do better what we have been trying to do?'" (p. 548). Hiebert and Carpenter (1992) stated, "Progress in achieving widespread implementation of curriculum programs stressing understanding depends on being able to document the outcomes of such programs" (p. 89). They declared, "In fact, assessment of understanding may depend on instruction and the language developed to talk about connections" (p. 89).

When implementing graphing calculator use in mathematics classes, the assessment issues involve both the learning of the calculator and the learning of the course objectives. More attention has been paid to the latter than the former. Many questions are unanswered at the present time. Most mathematics educators have beliefs about these relationships but there is little evidence to evaluate.

In the attempt to evaluate technology use on tests, there is some consideration regarding constructing appropriate tests. Harvey (1992) focused on whether test items are independent of calculator use. He distinguished between calculator-passive, calculator-neutral, and calculator-active testing. He wrote, "A *calculator-active test item* is an item that (a) contains data that can be usefully explored and manipulated using a calculator and (b) has been designed to require active calculator use" (p. 154). The importance of the decisions made by teachers regarding test construction cannot be underestimated. After all, Heid (1988) noted that students believe "that the test is the major determinant of what is important in mathematics"

(p. 711).

Romberg (1992) described a "plethora of procedures for gathering information from students: think-aloud interview procedures, performance tasks, projects" (p. 29). It may be necessary to design such activities specifically for assessing technology use. This is a difficult and time-consuming task for most mathematics educators. Decisions regarding projects, homework, group assignments, etc. require rethinking and rewriting with a technology focus. For example, Senk (1992) wrote that "making graphs becomes a tool for solving other problems, rather than an end in itself" (p. 129). Romberg noted, "Today, too many teachers are no longer trained in evaluation and lack confidence in their ability to judge student performance" (p. 35). Senk asked "What are the core content, processes, and beliefs?" (p. 131). Senk (1992) listed "issues faced by students and teachers"

> 1. mastering the technology itself
> 2. balancing exact and approximate answers, coping with multiple answers
> 3. putting the control of instruction and learning more with students than ever before
> 4. worrying about long term effects of less manipulative skill and more graphical representation on students' performance in subsequent courses. (p. 132)

With the incorporation of the graphing calculator into teaching and learning strategies, it may be important to clarify course objectives in the light of the demands that technology will make on instructors and students.

Learning Connections

Hiebert and Carpenter (1929) stated, "Connections that are weak and fragile may be useless in the face of conflicting or nonsupportive situations" (p. 69). When the goal of instruction is to develop understanding of a mathematical concept, are we creating for students a supportive environment for building connections by also placing before the student graphing calculators whose displays often require rather sophisticated interpretive skills? Will educators find ways to utilize the graphing calculator in order to challenge students to begin to see a more fully developed view of mathematics? Hiebert and Carpenter contended, "If the learner tries hard to fit a new idea, fact, or procedure into a current way of thinking, existing networks constrain the relationships that are created" (p. 70).

How do educators assist students in challenging the limits of such constraints? Perhaps the graphing calculator is the tool that can help develop many examples quickly so that an idea or procedure is no longer new to learners, but rather a part of their repertoire. Hiebert and Carpenter stated, "Well-rehearsed procedures capture a kind of mathematical power because they exploit the consistency and patterns in mathematical systems and guide the seemingly effortless solution of routine problems" (p. 78).

EDUCATIONAL CONCERNS

Heretofore, students have been presented, *fait accompli*, with graphical representations of functions in textbooks or on teacher-produced materials (chalkboard, handouts, etc.). These graphs being reasonably accurate, teachers are accustomed to advising students to carefully study them. Now, students themselves can produce graphs of functions with their own graphing calculators. We must question (1) whether students originate reliable representations with their graphing calculators, and (2) when they do, are they able to properly interpret what they see on the screen?

A major student difficulty is misunderstanding of the concept of order of operations and the impact it has on properly entering expressions in the calculator. Such entry errors may lead to inappropriate graphs with the student unaware of the mistaken outcome. Students who have found ways around some order-of-operation issues in entering numerical data (by using an enter or execute command before the end of the calculation or using pencil and paper to record intermediate results) are not able to use those same strategies when entering literal values. An equation of a rational function such as $f(x)=\dfrac{1}{x-2}$ is usually represented with the numerator stacked over the denominator and the fraction bar acting as a grouping symbol. [Note: Graphics boxes are used throughout this paper. In WordPerfect, the contents are not visible on the monitor except using the "View Document" feature.] Students are often unaware of the necessity to insert parentheses when entering some expressions. Even after instruction regarding the need to include parentheses, in some cases it takes a savvy student to transfer this information from situation to situation (e.g., $g(x)=\sqrt{x-5}, h(x)=e^{x+1}$).

If students are asked to generate graphs on the graphing calculator as a problem-solving tool, this requires that students learn to operate the calculator with appropriate insight regarding the information to be entered. Students are frequently unaware of the detail which must be mastered to enter some relatively simple-looking function equations properly. Some students operate under the misconception that if they enter a function into a graphing calculator, they will see a realistic representation of its graph. A student who is content as long as a graph appears too readily accepts what they see as the graph of the intended function. To be successful, students need to be able to interpret the screen display and make decisions about outcomes. Even when data is entered properly, are students aware that the displayed image may be incomplete or not revealing important details? Are students skilled in knowing when to investigate further to verify a suspicion that a display may be misleading or only partially illustrative of the salient features?

What does the student understand relative to what is on the screen?

Student use of graphing calculators is difficult to assess. When a student is not successful achieving a course objective, can we separate out the calculator issues which may be clouding the assessment?

CALCULATORS IN THE CLASSROOM

Beginning Fall Quarter, 1992, certain sections of mathematics at DeKalb College were designated in the schedule of classes as requiring either a TI-81, a Casio *fx*-7700, or other graphing calculator approved by the instructor. This paper reports on six such mathematics sections. The two researchers taught separately a calculator-designated Fall Quarter college algebra section and a business calculus section, a Winter Quarter college algebra and a business calculus section, and a Spring Quarter college algebra section and intermediate algebra section. Altogether, there were 159 students enrolled in these classes.

The students were generally representative of the usual enrollment in these courses. DeKalb College is a two-year institution with approximately 16,000 students on five campuses in metropolitan Atlanta. There is diversity of gender, race, national origin, and socio-economic status.

The two teachers were experienced trainers of graphing calculators use. They had together conducted workshops for college mathematics educators and secondary mathematics educators. They had used graphing calculators in a summer session for high school students. They had developed graphing calculator materials.

Both teachers began Fall Quarter with great expectations for the learning environment. They planned to teach the usage of graphing calculators as appropriate with the content along with some additional project assignments. They would make no significant changes in test items.

No student expressed concern over the cost of the graphing calculator. This may have been due in part to the arrangements made by the bookstore to rent graphing calculators for a fee of twenty-five dollars per quarter. While a number of students took advantage of this offer, more students purchased their own or were able to borrow one.

There seemed to be a level of excitement in the classroom at the beginning of the quarter that was usually not evident. Some students were operating under the belief that the machine held THE ANSWERS. Other students were just grateful that they would have some sort of assistance in the form of the graphing calculator. It wasn't long before students realized that retrieving the answers from the machine would require something more of them than simply punching some keys. Not only would they have to determine which keys to punch and when, but also how to interpret what they saw and when to dig a little further.

Neither researcher was satisfied with the outcome of the Fall Quarter classes. Instructors faced frustration, when despite the numerous classroom attempts to use the calculator to facilitate visual thinking, students would fail to see the interrelationship between the algebraic manipulation and the graphical interpretation. Though class retention rates seemed to be somewhat better than usual, the differences from previous quarters did not appear dramatic. The instructors also had to make important decisions regarding the use of class time to demonstrate some keystroking (a secondary priority) while in the midst of teaching a particular concept (a primary priority). The expectations of what could be accomplished by the instructor and assimilated by the student seemed to be not met in actuality.

As a consequence, the instructors planned to alter their approaches thereafter. Outside-of-class projects would be assigned for the purpose of detailing keystroking sequences. More student writing would be assigned to attempt to better involve students and train them to be more reflective regarding matters of their own learning. Collaborative work during class would be assessed with an eye to assessing calculator proficiency. Teachers would conduct one-on-one interviews with students as a means of assessing calculator proficiency as well as integration of calculator tasks with mathematics objectives concerning functions.

Calculator-Active Projects

Entering expressions appropriately contributes to some of the recurring problems students have with calculators. An early assignment was designed to highlight and emphasize the use of algebraic logic and the order of operations. Students were asked to compare their computations of arithmetic expressions done with paper-and-pencil to calculator results. One obvious advantage of using a graphing calculator is the opportunity for the student to see each entry as well as the output. Students could mentally evaluate $\frac{8+4}{2}$. When students would enter $8+4\div 2$, they were in a quandary as to why the calculator reported a 10 when they mentally calculated 6. Some students made the inference from such experiences that the calculator can't be trusted or that doing mathematics in your head was easier than using the technology. There was little acknowledgement that operator error was the difficulty and no understanding that, if the problem had instead been $\frac{8.396+4.27}{2.609}$, the same aspects of entering the numbers would be paramount. Furthermore, such students failed to see that the same requirements would hold for the calculator entry of literal values.

Students were asked to write about what they learned from this exercise. Some students' reflections were very pertinent while others revealed gross misconceptions. Their written responses included the following statements. [Note: All student writing quoted in this paper is unedited.]

Student 1: If you put the problem in the calculator wrong and don't know the answer in your head the calculator will miss guide you.

Student 2: The calculator does not always know order of operations eg. $8+4\div 2$ gives you 10 on the calculator but if $(8+4)\div 2$ is recorded, then it knows order of operations.

Student 3: Even if you do not know the answer your T.I. does. Whether you understand its system of operation or not it will give you an answer.

Student 4: I've learned that even the simplest (looking) problems may be a little tricky/thought provoking if not handled correctly. An answer may appear to be one thing naturally, but result to something totally different when working with a computer.

Student 5: I would have to say that the TI-81 calculator is close to a computer than a calculator to me. I learned that using this machine is more like programming a computer than punching buttons on a calculator. In that sense even though this machine has amazing power the user has to have the fundamental of algebra in order to use the machine properly. In my case it is going to take much more practice to master the TI-81.

Student 6: From the first project, I learned first of all - how to use the parentheses buttons to set off numbers. Then, I learned the 2nd button for using the (^) degree. I also learned to use the screen - its advantages of seeing many equations and answers at once and being able to use the insert button and the delete button.

Student 7: I learned that it seems easier to use than a regular calculator and shows your steps. The replay feature is also useful for checking. I learned that the calculator is not hard to use for simple equations and the fraction button is a great help. The calculator also aids in the harder problems by showing your work. The $\boxed{x^y}$ key makes the problem go quicker but I still like to work it out on my own and to use the calculator only in difficult situations.

Student 8: I must honestly say that I learned a lot from doing the Project I assignment. I learned how to roots. For example, 2 would be 2^4 ENTER. I never knew that I could set my own programs on a calculator. I didn't know that a calculator graph (in class exploring). I especially didn't know that could control the lightness and darkness of the screen. Boy, was I surprise. I really feel like I am in control, but this time, things have to be distinct and exact. it really makes you think about every step in operations. I can say I was familiar with the $\boxed{\texttt{ON}}$ button and the $\boxed{\texttt{2nd}}$ button. Overall, it will expand my knowledge about the calculator. It takes a little time. Like they say, practice makes perfect.

Student 9: In this exercise I learned that the calculator knows the order of operations better than I do.

Some of the comments were predictable. Some of them reveal beliefs that may be surprising. If the remarks of Student 1 reflect what he actually believed to be true, then is the calculator of any use to him at all? Student 2 appeared to have a misunderstanding regarding the concept of the order of operations altogether. More than one student expressed the notion that when the operator omits the necessary parentheses, it is the computer that doesn't know the order of operations rather than an error of input. The remarks of student 3 leads one to wonder if the student's notion of success is getting AN answer. Among students, there is a naivete regarding what the calculator brings and does not bring to a problem solving activity. For some students, there is a belief evident that if a calculator comes up with an answer, it must be correct. While other students, including Student 4, seem to believe that what an answer IS is dependent on whether a calculator was utilized or not. Many students expressed the idea that to input an expression may require thought and is not a haphazard process. Remarks (i.e., Students 6, 7, & 8) also reflected that the project introduced students to some of the features of their machine, one of the instructor's intended goals. The pithy statement made by Student 9 summarized what may be more common than some would like to admit.

Educational strategy was impacted by the use of graphing calculators. Instructors were able to devise projects calling for students to produce many graphs in order to see some emerging patterns. One project goal was to prime students before classtime in order to make better use of the contact minutes in class. Another aim was to create a more student-centered environment and get away from what Dossey (1992) called the "broadcast mode" of teaching. Asking students to participate more actively in articulating the patterns and forming generalizations is consistent with the National Council of Teachers of Mathematics standards (NCTM, 1989) and a strategy which leaves the student with an example of a means of investigating functions on his own.

Another project was given to students to aid in developing the concept of exponential functions. The project was assigned before any lecture and classroom activities dealt with exponential functions. The project was designed to illustrate the general appearance of exponential functions and the relationship between modifying the equation of an exponential function and the resulting translation or reflection of the associated graph. The notion of translation and reflection had been done some weeks prior with some standard functions (parabola, hyperbola, square root, cubic) but had not yet been discussed relative to exponential functions.

The instructors observed that when a student completed the assigned work in advance of a lesson as requested, the classroom development of a concept seemed to go more smoothly and included more student participation. For the students who completed the project only in part or not at all, classroom discussion and development of concepts were dramatically less meaningful, if not extremely confusing.

After the students completed the project which generated graphs of polynomial, radical, or rational functions not yet discussed in class, they were asked to reflect on their works. Excerpts of their comments follow.

Student 10: The project was interesting in that all was required was to put the equations in the graphing calculator and the graphs come out great. I found it more fun than writing out a table in order to get the graph. The graphs weren't complicated and easy to graph. I had few problems with the exercise, the main one was looking at a graph and find the equation. It was the main part that gave me problems. The graphing calculator makes it easier for me to solve the problem.

Student 11: Using the casio to graph was a spiritual experience! Being able to graph without having to do the math is the greatest thing since sliced bread.

Student 12: One of the most important things that I learned from this project is recognizing a function just by looking at its equation. In another case, I have also learned how to [not legible] the equation by looking at the graph. Most of the functions follow certain trends. I mean, the same kind of equation you have, the same kind of graph you will have. This project has given us more ability, when we are dealing with functions.

Student 13: As I was working on project 3, I remembered a lot of the graphing that I had done in high school algebra, so it was pretty familiar. The TI-81, however, produced some hideous pictures of the graphs, especially the rational ones. The graph and the asymptotes seemed to be blurred together, but if I zoomed in on a small part of the graph the picture became clear enough for me to recognize the pattern and draw the graph. I could also recognize where the asymptotes should be based on the equation, so the horrible pictures from the calculator didn't cause too much trouble.

For many of the students who responded as did Students 10 and 11, there were serious errors in the graphs they turned in. Such students accepted at face value whatever they copied off the display. The belief that they didn't have "to do the math" was detrimental to creating proper graphs.

A few of the students responded like Student 12 who discovered some relationships of families of functions in the exercise. But it was a rare student who articulated calculator concerns as well as Student 13 did. She proved to be the exception rather than the norm.

Class projects were devised not only to acquaint students with families of functions. Instructors also hoped that such exercises would provide the students with a model of a strategy of investigation, however the instructors observed very few times a student or group of students self-initiated investigations with the graphing calculator.

Classroom Observations

Another classroom strategy proved to be a learning situation for both students and instructor. The class was divided into small groups of three or four students in each group and a problem was projected on the overhead. None of the problems had been formally introduced in class yet or assigned as homework. The instructor sat with each group and observed for about ten minutes. The instructor recorded notes of student interchanges regarding problem solving and calculator strategies and did not interact with the students.

Students who had no previous exposure to graphics calculators displayed great differences in their ability to manipulate the calculator. Though there was cooperation among students to show others how to execute some particular keystrokes, it appeared that the extent to which a student employed the calculator outside the classroom might account in part for the wide range of skill levels demonstrated.

It was noticeable that students didn't always think critically as to when it would be appropriate or efficient to use the calculator vs. mental or paper-and-pencil skills. Sometimes students would go out of their way to try and use the calculator. One group was asked to get the decimal approximation of π^e to the nearest hundredth. The group members spent some minutes considering how to adjust the mode of the calculator in order to have the calculator round appropriately, instead of having the calculator evaluate the expression and then round by inspection. On the other hand, when a student in this same group was asked to tell everything she knew about $y = e^{x-2}$, she spent eight minutes constructing a chart with many ordered pairs, plotting only one of the points and labeling it, but creating no more of the sketch of the graph. The calculator remained untouched on her desk.

Student Interviews

Researchers worked to gain insights into student thinking and understanding of calculator use by conducting interviews. Students were told that this project would be individual fifteen-minute visits to the instructor's office for the purpose of using the graphing calculator for activities similar to those conducted throughout the quarter. To alleviate concerns about talking alone with the teacher, it was announced that participation merited full grade for the project. By the time of the interview, students had been required to work regularly for about nine weeks with their graphing calculators. The two teachers implemented these interviews with three classes. Fifty-five out of sixty-five students participated. The others were excused because of schedule conflicts.

It was decided that the interview was not to be a vehicle of instruction of students. The instructors wanted the students to do as much of the talking as possible. It was difficult on occasion not to jump in and take on the familiar role of telling or to let the student state incorrect information without trying to remediate on the spot. The fifteen-minute time span proved to be rather short for extended conversations.

Upon beginning the interview, a student was given a sheet of paper which listed up to three equations in functional notation across the top of the sheet. The functions were of the same variety that had been examined in class in prior weeks. Throughout the session, the instructor took notes regarding observations and student comments. In addition, any scratch work done by students was kept by the instructor. The instructor would point to an equation and ask students to tell what they could about it. Each student was not asked the exact same questions since the dialogue was in part driven by the student's response. However, many questions were repeated from one interview to another. When students were asked to tell about a particular function, some would ask, "Do you want me to use the calculator?" Some students would immediately turn to paper and pencil, ignoring the calculator.

The following are summaries of some case studies of interviews conducted with college algebra students.

CRYSTAL

Crystal was able to produce a calculator graph of the function $f(x) = \frac{1}{x+2}$. When asked to discuss the graph, she mentioned the asymptotes, domain, and y-intercepts. When asked what f(2) was, by inspection of the function equation, she replied, "One-fourth." When asked how that related to the graph, she said she was not sure but that it "might be" one of the points. When asked which point she thought it might be, she answered, "(1/4, 1/2)." She was unable to explain where the ordinate came from. Her response appeared to the instructor to originate from her visual approximation of the graphics display.

When asked to graph $g(x) = e^{x-9}$, Crystal entered the function and successfully adjusted the range of the calculator to get a more revealing viewing window. When asked if f(x) and g(x) had any points in common, Crystal said, "Maybe (8,1/2)." When asked if she could clarify that with her calculator, she used the zoom feature and adjusted her guess to (6,1/2). There was no initiative on her part to substitute values into either equation to verify algebraically or to continue to use the zoom feature to refine the guess.

When Crystal was asked to evaluate $g(\pi)$, she attempted to use the graphing feature to produce the graph of $y = e^{\pi - 9}$. There seemed to be a general lack of understanding regarding relationships between a point on a graph, a series of points on a graph, and a functional value associated with some number. Many students seem to have misconceptions about whether it is appropriate to complete a computation or consider a graph.

DOUG

Doug was asked to graph $f(x) = \frac{1}{x+2}$. He attempted to enter the function but did not include any parentheses. When asked if he could somehow check out if the display properly represented what he expected to see, he suggested he could plug in numbers for x and "work it out." However, he did not do that. He volunteered that he wondered about asymptotes. He said, "I can't remember what the rule is. I know -2 would make it zero but I don't remember if that's vertical or horizontal." Again he repeated that he couldn't remember the rule. He added that he didn't remember for which one but that for one of the asymptotes one would look at the degrees.

Doug was then asked to graph $g(x) = \frac{1}{x} + 2$. He did, and he reported, "It's the same graph." He was asked if he would expect that to be so. "Yeah," was his immediate answer. These relatively simple rational functions had been prevalent in classroom examples, group work, and homework assignments for weeks. Perhaps Doug's quick reply was a display of his anxiety over meeting in an interview situation. How can one measure the degree to which Doug's nervousness might account in part for his not seeming to be aware of or question the difficulties? Doug not only displayed inadequate facility in entering a simple rational function, but there was no evidence that the concept of translations of rational functions was part of his working knowledge.

MARK

After graphing $f(x) = \frac{1}{x+2}$, Mark was asked to name the y-intercept. Mark used his calculator to store the value of zero for x and to evaluate the expression $\frac{1}{x+2}$. For this particular example, it would have been easier to achieve the computation mentally rather than to complete all the keypunching necessary. When asked about the value of f(3), he did complete that mentally and responded "1/5" with a question in his voice.

When asked about the intersection of two functions, Mark was able to graph both f(x) and $g(x) = e^{x-9}$. When asked if they touched, he pointed on his display screen to the appropriate area. He was able to skillfully use the zoom/box feature and clarify the point of intersection. Mark was clearly in the top fourth of the class in ability to demonstrate the calculator skills useful in college algebra.

YUMI

Yumi was asked to graph $f(x) = e^{x-9}$. She entered it without the necessary parentheses and didn't seem to be aware of the matter. She was asked to evaluate $f(\pi)$. In omitting the necessary parentheses, she arrived at an erroneous result. Then she was asked to evaluate e^{x-9}. When asked, "What do you notice?" She responded that they were the same. The instructor asked, "Do you expect them to be the same?" She didn't answer. When asked further, "What do you think?" She responded, "I need parentheses." It was reassuring to witness Yumi's ability to construct the comparison, with some instructor questioning, and recover from the omission. When asked to consider $h(x) = \frac{1}{x+2}$ as well as h(10), Yumi was one of only a few students who was able to indicate a location on the graph which was appropriate.

AMY

When Amy successfully had the machine graph $f(x) = \frac{1}{x+3}$ and was asked to describe the graph, she said that it crossed the y-axis. When asked where that happened, she said, "A little above zero." When asked if she could use the machine and be more precise she said, "I don't know how. Claudia [Note: Claudia was a fellow student who sat near her in class.] tells me how. I think I could push TRACE." When asked to go ahead and try that, she again reiterated that she didn't know how and was unable to proceed.

When asked to graph $h(x) = e^{x-10}$, she was not able to enter the function and asked, "Why isn't anything working? It works in class." It seemed probable that in class she got enough coaching from others to feel as if she could manipulate the calculator but actually did little on her own to reinforce these skills.

Educators will likely continue to be faced with students who don't complete or attempt the outside work assigned or the type of study skills that are suggested. Is Amy's case a technology style example of the popular misconception that work done in class, without practice outside of class, is sufficient? How long will a student such as Amy be in mystery as to why things aren't working? If the calculator becomes one more item in the pile of "things to do but left undone," it seems unlikely to have significant benefit for that student.

YVONNE

Yvonne was asked to graph $f(x)=\dfrac{1}{x+2}$ and tell about the graph. She entered it into the machine successfully and indicated that it had an asymptote "between here," indicating with her finger on the window display. She said that it will have a y-intercept and another asymptote at y=0. When asked what the y-intercept was, at first she said "0." Then she went to pencil and paper and substituted a zero for "x" in the equation, and when she got 1/2, she seemed surprised. Without any coaching or questioning from the instructor, she began to zoom in on the graph.

When asked about the value of f(2), she again went to paper and pencil and got 1/4. She asked, "Is it a point?" The instructor responded, "What do you think?" She said, "It can't be the y-intercept." When asked whether it was the x-intercept, she said she was not sure. When asked to find f(5/2), she could compute it with paper and pencil. But, when asked, "Are these points on the graph?", she could only say that if you want to have points, let x equal some number.

There seems to be a lack of integration between what the students can mechanically produce with paper and pencil and with how they can relate the results to the graphical interpretation. Had Yvonne been asked what the x-intercept was, she likely would have been able to arrive at the result with paper and pencil as well as point to the appropriate area of a displayed graph, but when asked if a particular point is the x-intercept she was unable to put those other skills into play. The mechanics of arriving at an x-intercept algebraically or indicating on a graph the location of an x-intercept is familiar to many students, Yvonne included. She had indicated such skills successfully on class tests. However, there is a lack of real comprehension of the concepts and connectedness among related concepts.

ZACK

Zack, who forgot to come for the first two appointments he had made for the interview, arrived without his calculator. When he was asked to graph $f(x)=\dfrac{1}{x+2}$, he asked to borrow the instructor's calculator. He was handed a Casio *fx*-7700, since that was the same model he owned. Upon its previous use, the zoom feature had been used so the range settings were unusual. Zack struggled to adjust this saying he could do the graphing task by plugging in points but that he didn't want to write. He reported that he had

had trouble adjusting his range before. After much punching, he got to a standard viewing window. It was disconcerting to see such a necessary graphing calculator skill was so difficult for Zack to achieve.

When Zack was asked about the graph, he volunteered that, "There is an asymptote at 0 and at -2." When asked about the domain, he said he was not sure about it but that it should be "from 1." Zack did not appear to consult his graphics display to come up with this answer. Was it that Zack had a ill-conceived notion of the concept of domain? Without an understanding for the concept, would seeing any image on the display screen have been of assistance to Zack?

When asked to graph $y = e^x - 9$, he could not properly enter the equation. He said he wanted to tell the calculator not to raise the 9. However, he was not able to even produce appropriate keystrokes to enter $y = e^x$. Zack suggested that he could get the decimal approximation of "e" and then subtract "9." He evaluated e^x on the machine, seemingly unaware that some previously stored value was replacing "x." Zack's calculator keystrokes produced 8.674, he subtracted 9 and then graphed y= - 0.326. When the graph of a straight line appeared, he suggested that it must be an asymptote. How widespread is the misconception that a particular value for e^x is interchangeable with the function $y = e^x$? Does the college algebra student wonder how the two relate to each other?

When asked what $f(\pi)$ has to do with the graph of f(x), Zack said his intuition was that it appeared to define an asymptote. The notion of an asymptote is powerfully illustrated graphically, but for Zack this was unavailable since he was unable to graph the function with a calculator in the first place.

The following students, in a business calculus class, were asked questions about functions, limits, and tangent lines.

KEVIN

Kevin was asked to graph $f(x) = \frac{x^2 + 10x - 24}{x - 2}$. He made appropriate use of parentheses in entering the expression. The resulting graphics display was a small portion of a line in the upper portion of the second quadrant. Asked if he thought he had been successful in producing the graph, he said "Yes" without making any further comment on the appearance of the display.

Then he was asked to evaluate $\lim_{x \to 2} f(x)$. He said that "he was trying to remember how to do it" and began to write and "use the formula." After several minutes, he said, "That's wrong." Asked if the calculator might help, he readily began to trace. He stopped when the calculator display read "x = 2 y = ." He interpreted that as, "From what this says, there's no y for 2. 2 makes it undefined." Despite this analysis of the situation, he never did determine if the function had a limit as x approaches 2.

Then Kevin was presented $\lim_{x \to \infty} f(x)$ to consider. He responded inconclusively, "Take 2 over 1 means that . . ." Again, it was suggested that the calculator might help him. He gained confidence as he looked at the screen. "It keeps going," he said. "It has no limit."

Kevin, like some of the other students interviewed, had had prior experience with graphing calculators. His high school mathematics teachers had occasionally provided a class set. The previous quarter at DeKalb, he was using his own TI-81. However, he stated that he had learned things about his calculator in this class that he had not known before. This was also true of the other students with prior graphing calculator experiences. Questions naturally arise about the required quantity and extent of instruction in mathematics technology necessary.

NICOLE

When given the function $f(x)=\dfrac{x^2+10x-24}{x-2}$ to graph, Nicole typed in the statement without any parentheses and was satisfied with the result. After some prompting, she set out to reenter the expression completely. She admitted when questioned, that she did not know how to use the insert command. When the small line segment appeared in the second quadrant, she said, "Now you change the range. I will try to figure out something." At this point she set a minimum value for y of 10 and a maximum value of y for -5. When the error message appeared, this led to a discussion of minimum less than maximum. At the successful resolution of this dilemma, more of the graph was revealed. She said, "It gave us the left side, now it is giving us the right side."

Nicole was now asked to evaluate $\lim_{x \to 2} f(x)$. She asked, "Is it going to infinity? It is going to infinity." Then she was asked about $\lim_{x \to \infty} f(x)$. She queried, "Where would I start? I don't know. It is going to infinity. It looks easy but is hard." She was well aware of her mathematical deficiencies. The basic mistakes in both content and technology that Nicole evidenced were painful to witness.

EDILYNN

When Edilynn was also given the function $f(x) = \frac{x^2 + 10x - 24}{x - 2}$, she used the fraction key on her Casio as well as the appropriate parentheses to enter the function. When nothing appeared on the screen, she was unperturbed. "It won't show up," she said. "I used the default range." Then she selected a viewing window which used [-10, 10] for both x and y. After a <u>long</u> silence viewing the result, she said, "I guess it's all right. It doesn't look right. I was expecting a parabola. It looks like a straight line." She made no more comments on her dilemma.

She was now asked to state the limit of f(x) as x approaches 2. Then there was another <u>long</u> silence while she looked at the graph. "They have different exponents," she said. "Do they have an asymptote? Won't you have a zero in the denominator? Do you factor?" At this point, she did write down $\frac{(x-6)(x-4)}{x-2}$. "It looks better now," she decided. When it was pointed out that +24 would be the product of -6 and -4, she redid the calculations manually and rewrote the factors properly. She stated, "You cancel out a factor. It's doing that thing again!" She seemed satisfied with her labors and never returned to the question of the limit.

While Edilynn was reasonably competent with calculator skills, she and other students appeared to lack focus on problems. Despite voicing perplexity on whether the function should be a parabola or a line, she abandoned the question too easily. Is it too demanding a proposition to reconcile dissonant views? Her efforts to produce correct factors of the rational expression overshadowed the search for a limit. When she was not sure of herself about limits, was it safer to take refuge in the algebraic procedure and avoid further exposure of her lack of knowledge?

SULTAN

The graph of function $f(x) = \frac{15x-6}{x+1}$ is outside the standard viewing window of both the TI-81 and the Casio. Sultan never hesitated when asked to generate the graph of this function. When a complete graph of the rational function appeared, he was asked to reveal the range settings. They were [-20, 20] for both x and y. It transpired that the calculator had been set for these intervals when he got it. He had never changed the range since.

In describing the graph, he said, "I see a vertical line x = -1. What is this called? It comes from left and goes right." When asked to evaluate $\lim_{x \to 1} f(x)$, he said, "I don't know. I can solve it. It is 9/2." Then he was asked to evaluate $\lim_{x \to -1} f(x)$. He responded, "It is 21/0 which is getting smaller and smaller." Then he looked at the screen. "This one is going bigger. It goes to y-axis." When Sultan was asked to state $\lim_{x \to \infty} f(x)$, there was a long silence. He said, "I don't know." Then he added, "That's going to be 1."

When Sultan was asked if the problems he had been asked had been hard, he responded that they were not. "I can put the function in," he said, "but I can't get the points." At least he was honest about the limitations of his expertise.

Student Reflections

After the last of the interviews in one class were conducted, on the last day of the quarter, the teacher put each of the three demonstration problems on the board. In a very matter-of-fact manner, the teacher analyzed them. Deliberately, there was no reference to the students' efforts on these problems. Then the students were asked to write about the experience of the project. Following are excerpts from these statements.

> When I left the office after doing project four, I realized almost immediately that I made a couple mistakes. Sometimes I rely to heavily on the calculator and not on my own brain. I have the attitude that the calculator is always right. Truthfully it also makes mistakes

because it is programmed by humans and can only do limited things. I learned I also need to try and work things out by hand sometimes or at least double check the graphs. Also I have been getting very nervous and second guessing myself. In the homework of which I do about 75%-80%, I think I understand it all. I just cannot remember which formulas or operations are for what problems.

This project is probably the one thing that helped me the most this quarter. My problem was e^{x+1}, and to find the equation of a line tangent to x = 2. This problem helped me to find the slope of the equation by plugging x = 2 in the main equation. This problem also helped me to understand that the equation of the line tangent and the main problem, if graphed will intersect at x = 2. This problem refreshed all my memories about the problems we had worked in the first half of this quarter. This project was a major help. Thank you.

The problem I did was one we had done earlier in the quarter and I had trouble remembering what to do. You reminded me of the key terms which I had forgotten, such as taking the limit of a function. I had forgotten that was simply taking the derivative of the function.

The problem that I was assigned was e^{x+1}. I knew how to do similar problems to this but I was unaware what the words "tangent" to actually meant. Until I worked on this problem I did not know what it meant, and seeing it displayed on the graph re-inforced it. This problem also reinforced how to solve a problem using the equation $y - y_1 = m(x - x_1)$. It also reinforced using the slope. Overall I think taking a quiz like this is a good idea but it's a little nerve racking. I think I can concentrate more when a problem is given to me rather than discussing it.

I regret that I was unable to do the project [Note: This student did not participate because of schedule conflicts.] because when I attempt problems on my own they always seem ten times harder than when you explain them on the board. I can follow along when you do the problems, yet I have trouble doing them on my own.

What I learn from the project is that the graph calculator can help you do the problem. But, you cannot depend on the calculator to give you all the answer. I learn in class about the problem, which I had forgot was that you could put the number in to the problem to get you answer for when the number goes to the lim what it will do. The problem in class also reminded me about how special e^x is. After I left your office, I was happy that I knew how to work another operation key on my TI, which was the trace key.

In Project 4, I was given a problem $\frac{15x-6}{x+1}$. The questions that were asked to me were not very difficult. First, I was very nervous when I came to your office, but after I answered one or two questions I felt comfortable. I think by doing this project I gained more confidence in myself. When you gave the answers for those questions in class, I think most of my answers were right. I think this is a very good idea of giving a project like this because students gain more confidence by coming to teachers office and answer the questions.

I enjoyed working the problems in a one to one basis but I was a bit nervous. After reviewing the problem in the board, I understand it very well. To be honest, I could have do the problem if you gave to me to work out. When you ask questions back and forth, it was a harder problem to do. I know now that you cannot expect the calculator to do the work for you. You need to think through your problems first. I did not expect to do an actual problem. I thought you would quiz us on the calculator itself. Showing us on the board today gave me a better understanding in going to do the problem. It is always better visual than of course learning how.

When I started working on my first problem I went to Y= and typed in x^2 + 10x - 24/(x - 2). I would usually mess with the keys for a bit before typing the actual problem, but that day I showed you the wrong answer. I should have typed (x^2 + 10x - 24/(x - 2). After realizing that mistake, I then went on to finish the problem by answering the questions you asked about the function. I felt good about the problem.

There are no particular problems for me in this course with you. Every problem makes me nervous and force me to have a flexible idea to understand it. I want to say that it is very convenient to learn math person to person than work out myself with book.

When I left your office I can say that I felt good about the problem. However, today in class I realized there was one or two minor details that I left out. These details can make a big difference on passing or failing the final exam.

After I visited you in your office, I realized that the question you asked me I need to explain the graph and its function. When I left your office, I went to the math lab and worked only with my calculator. It was very interesting. I have learnt from these equation how to adjust my range and how to work on graphing.

Despite the teachers' plan that the interview sessions be an opportunity for them to listen to students and gain insight into student thinking, many students reported the interview session to be a learning experience for themselves. Students recognized either at that time or upon later reflection that there were content matters or technology matters which they were handling inadequately. Thus, the time invested in the interviews by both teachers and students was mutually productive.

When students were asked if they would recommend that a friend take a course requiring a graphing calculator, the response was overwhelmingly affirmative. Their verbal and written statements indicated a belief that graphing calculators had helped them in the course. Students reported the efficiency of using the calculator to evaluate functions as very beneficial to their work. Also, many students stated that they used the graphics display to compare with student-constructed graphs. A number of students recounted working out a test problem first with pencil and paper and then checking themselves with the calculator.

Results

The interviews provided instructors the opportunity to see a side of student activity and student thinking that is not apparent in classroom interaction. The challenge however is how to respond, with these glimpses in mind, to better assist students in concept formation and to achieve the course objectives. Students revealed deficiencies of mathematics knowledge, difficulties with technology, and inability to reconcile inconsistencies of technology and mathematics content. The interviews were productive from the standpoint of uncovering the massive lack of integration between the paper-and-pencil algorithmic tasks which could be successfully performed and the interpretive tasks which need to be accomplished to arrive at consistent results from the graphing calculator.

In similar fashion, the classroom observations and projects were illustrative for instructors regarding student learning activities. In addition, the projects were useful teaching strategies involving introduction to particular keystroking patterns as well as guided discovery to enhance concept formation. For the instructors, the development of such projects was an additional feature of course preparation. The project work was worthwhile insofar as it allowed the student greater participation in the learning process.

Hiebert and Carpenter (1992) stated, "Memory, if viewed as a reconstructive process, involves the same cognitive activity as understanding; constructing connections between representations of new knowledge and existing knowledge" (p. 75). Enriching the classroom with graphing calculators does offer a student the possibility of greater ease in visualization, yet the successful integration of graphing calculators may still depend on the ability of students to integrate calculator skills as well as newly introduced mathematical concepts to their own knowledge bank.

SUMMARY

Mathematics educators have called for the integration of graphing calculators into the curriculum. Many teachers believe that the use of graphing calculators will promote visualization skills for mathematics and serve as a resource for problem solving.

Effective classroom procedures for attaining those goals are evolving. Students are now being asked to remember keystroke sequences and new modes of investigating problems in order to thrive in a technology rich classroom. The challenge to the mathematics educator is to determine instructional patterns which promote this development.

Requiring graphing calculators is a demanding proposition. These demands may not be readily apparent to either the user or to the teacher.

While there is the potential for enhancement and inquiry about mathematics topics, there are also concerns regarding acclimating students to a different learning environment. Since technology use in mathematics education these days appears to be a given rather than a potential factor, methods of classroom integration are now required. The challenge is worthwhile, however, and worthy of pursuit.

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