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Teaching Probability to Prospective Elementary Teachers Using a Constructivist Model of Instruction

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INTRODUCTION

This paper is a report of a study conducted with preservice elementary teachers at the University of Wyoming during the summer of 1993. The study had two purposes: (1) to observe the effectiveness of using a constructivist approach in teaching mathematics to preservice elementary teachers, and (2) to focus on teaching probability using a constructivist approach. The study was conducted by one instructor in one class, The Theory of Arithmetic II, a required mathematics class for preservice elementary teachers.

The instructor had taught this course a number of times in the past using a traditional lecture approach and had become concerned that many prospective elementary teachers viewed mathematics as a fragmented collection of rules, algorithms, and definitions which they memorized just to fulfill the requirements of a mathematics class. For many students there was plenty of information but little apparent understanding (Steffe, 1990). This is a critical distinction for two reasons. First, the students in this class will be teaching our children mathematics in the future. If they do not understand what they have learned, they will teach mathematics in much the same way as they have learned it. If there are misconceptions or misunderstandings, they may be transmitted to their future students. Secondly, if these students understand a key concept or why an algorithm works, they will have a better chance of remembering it longer and it will become a useful tool in learning more mathematics. In previous classes the instructor had slowed down the pace of the class to focus on critical concepts and had used small group discussion as a means of reinforcing the understanding of these concepts. This idea had proved fruitful and had suggested teaching the entire class using a constructivist approach.

One of the major content areas of this class is probability. One of the goals of this paper is to examine whether probability can be taught effectively using a constructivist approach. Probability was chosen as the specific content area for a number of reasons. It lends itself well to a problem solving approach, since there are many fascinating, counterintuitive probability problems such as the "birthday problem" which can be used in a classroom. Students have little mathematical background in probability and, thus, must construct their own knowledge about the subject and not just use a formula, like that for the area of a circle, that they have learned in another class. Another merit to teaching probability in a constructivist manner is that in many problems students can use a simulation or a hands–on, empirical approach to a problem to confirm or refute their theoretical ideas. For example, in this class the students vociferously discussed the probability of having one boy and one girl in a family of exactly two children. Some argued for 1/2 and some for 1/3? It was determined to toss two coins a large number of times and record the results to examine their theory.

METHOD

The idea for this paper came from a number of successful studies conducted with elementary students which used a constructivist approach to mathematics instruction (Carpenter, Fennema, Peterson, Chiang, & Loef (1989); Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz (1991); Cobb, Wood, Yackel, and Perlwitz (1992); Maher and Alston (1990)). Wheatley provides an excellent description of the method used in this study (Wheatley, 1991). "Problem centered learning", as he terms it, has three components: tasks, groups, and sharing. To prepare for a class, a teacher chooses tasks which have a high probability of being problematical for students. The instructor of this class chose some problems from the required textbook for the class, <u>Modern Mathematics</u>, by Ruric E. Wheeler, considered resource material for the students, chose other problems from various other sources, made up some problems himself, and used problems proposed by the students. The students in this class had a wide range of mathematical abilities and backgrounds, so problems were chosen which would be problematical for a majority of students. This was sometimes difficult, since, for a fair number of students in this class, finding the hypotenuse of an isosceles right triangle of side 1 was very problematical. Secondly, the students work on these problems in small groups. In this class the students formed their own groups on the first day of class and stayed with those same groups throughout the class. There were nine groups in the class, each with four students.

Finally, the class is assembled as a whole for a period of sharing, when groups present their solutions to the class, not the teacher, for discussion. In this class the students were required to present their solutions in their group standing up in front of the class. They were required to explain to the whole class how they had thought about the problem, what difficulties they had faced when solving the problem, what their answer was, and how they might teach it to children. Each group, after the presentation of a problem, had to ask for agreement and disagreement from the other groups on its solution. If another group disagreed, it had to be willing to suggest and justify another solution, generally done with politeness and sensitivity to the first group. When there was disagreement among the groups, the instructor did not mediate but allowed the class to reach a consensus. In these discussions the instructor was not a judge but was a facilitator whose goal was to ensure that the discussion proceeded along orderly lines and that students were engaged in constructing for themselves mathematics which was consonant with sound mathematical principles. It might be mentioned that at one time every group had solved the same problem incorrectly, obtaining the same wrong answer, so that the instructor had to assume a very active role for that brief period.

OBSERVATIONS

The instructor observed strengths and weaknesses of teaching this class with a constructivist approach instead of the traditional lecture format. The greatest strength of the method was that it engendered in the students a more energetic, enthusiastic, and active approach to mathematics. They were doing mathematics rather than watching mathematics done. In a recent conversation a mathematics professor had claimed that students liked to see a masterpiece being painted by the professor on the blackboard. They did not want to see their own crude brush strokes on the canvas. In this class, however, students were almost always engaged in their own crude brush strokes. At one time during the class, while the students were working on solving problems in their small groups, the instructor attempted to stop their work and proceed to the large group discussion time. The students were so actively engaged in discussing their solutions to the problems that they totally ignored the announcement a couple of times. Since classroom control was never an issue, the instructor was delighted to witness such enthusiasm for doing mathematics.

Another benefit to using a constructivist approach was that the students became more independent. They began to develop autonomy in justifying and defending their solutions to problems. In traditional mathematics classes the two sources for authority are the teacher and the answers in the back of the textbook. The student often does not persist in trying to solve a problem by his or her own methods but, instead, seeks help from the teacher at the first sign of trouble. The problem of teacher dependence goes even deeper when it comes to preservice elementary teachers. In the future, when they are working with children, they will have to be able to give reasonable answers to the childrens' questions. This demands their own independence. They must be able to do mathematics on their own. This problem of teacher dependence is usually compounded by traditional means of assessment. Often in a mathematics exam credit is given for correct answers and for correct method. The student's goal is, first of all, to get the correct answer. This often means finding the formula in the textbook that seems to correspond with the given problem, then using the numerical data in the problem until the answer at the back of the book is obtained. If there is no answer to a particular problem, there is often a similar example in the text or a similar problem with an answer. Such an approach to doing mathematics leads to such disturbing questions from the students as, "Do I add or multiply the numbers given in the problem to get the correct probability?" This is especially disconcerting when the sum of the probabilities in question exceeds one.

The student's goal is also to try to understand the teacher's method for solving a problem. However, the teacher's method for solving a problem is often the most elegant, sophisticated, concise, and abstract way of solving a problem, whereas the student's own method is pedestrian, somewhat inefficient, and concrete. The teacher has often distilled his or her method of solving a problem into a formula, the highest level and most abstract representation of the solution to the problem. The teacher in his or her career in mathematics has seen the concrete side of the problem many times but the student has not. A problem given in this class may serve as an example: A line is given with a number of points labelled on it. The student is asked to name as many line segments as possible. Since, at this point in the class, the students supposedly have studied permutations and combinations, a teacher may solve the problem in a very elegant manner by calculating combinations of the number of points given on the line taken two at a time. It was the experience of the instructor, however, that many of the students in the class could not produce a systematic, organized approach to listing all the possible line segments but, instead, proceeded in a somewhat random manner listing segments until it appeared they had them all. There could be a vast difference between the way a teacher might attempt to solve the problem and the way students will understand the problem.

An advantage of a constructivist approach to teaching prospective elementary teachers over direct instruction is that it provides an opportunity for the students in the class to develop skills valuable to them in the elementary classroom. Many students in this class had little confidence in their ability at mathematics and were terrified at the possibility of solving a problem on the blackboard or making an oral presentation to the class. For many of them, in their past such experiences had been humiliating and embarrassing, often episodes in a mathematics class in which they were publicly ridiculed or chastised by the teacher.

Throughout this class the instructor provided and reinforced guidelines for group presentations of problems which were believed to help reduce the stress associated with talking about mathematics in front of a class. The first guideline was that any approach to solving a problem was valid and potentially productive. This was to decrease student dependence on the instructor and textbook and give students more freedom to express their thoughts. Another guideline was that any method of solving a problem must be able to be justified. This meant that, if students used a particular method for solving a problem, they had to be able to explain and justify what they had done. If students used a formula, they had to demonstrate an understanding of the mathematical concepts behind the formula. For example, if students solved a problem for calculating a permutation, they could not simply explain to the class that they had computed various factorials on their calculator, but rather had to demonstrate the logic behind the counting of objects expressed by a permutation. A third guideline was that it was acceptable for a group, when giving a presentation of its solution to a problem to the class as a whole, to summarize how far it had progressed in attempting to solve the problem. This entailed a discussion of their interpretation of the problem, the information known for certain about the problem, the attempts to solve it, and the interaction among the group members. This guideline validated honest mathematical activity and perseverance, deemphasizing mathematics as simply finding the correct answer. It also forced each group to be ready to communicate the results of its work and not just announce that it did not know what to do. A fourth guideline was that each group was encouraged to consider and comment upon the pedagogical implications of the problem for elementary education, creative ways to teach the concept or solve the problem, misconceptions students might have, uses of manipulatives to model the problem, real life applications of the material, and other similar matters of practical interest to a prospective elementary teacher.

Using a constructivist teaching approach in this class was a start at helping the students gain skills necessary for becoming good elementary mathematics teachers. They appeared to gain confidence in doing mathematics on their own and in communicating their own ideas to fellow students within their own groups and to the class as a whole. They also appeared to lose some of the anxiety or fear of mathematics by having their own ideas about problems recognized as valid and useful and by having many nonthreatening opportunities to enjoy success in mathematics, a rare occurrence for many of these students in the past. The students seemed to make substantial progress toward the five general goals articulated in the NCTM Standards (NCTM, 1989): (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4), that they learn to communicate mathematically, and (5) that they learn to reason mathematically. In sum, this class provided the students with an environment in which to do mathematics very much in the spirit of the Standards, so that the students of this class gained more than just knowledge and information.

The instructor found one of the most difficult tasks facing a teacher in a constructivist classroom is defining his or her role in a new way, that of a facilitator, one who ensures that the classroom community is functioning smoothly and constructing knowledge that is mathematically sound, not that of the sole authority and dispenser of knowledge. It was sometimes difficult not to just tell students exactly what to do to solve a problem or surmount a difficulty. This was especially true when there was a large group discussion. Often the instructor resisted just jumping into the discussion to cut the Gordian knot with one quick explanation, clarification, or comment. Instead, he assumed that the time spent in whole class discussion about the individual group's methods of solving problems was productive in helping students to better understand the mathematics involved. There was also some discomfort in redefining the teacher's responsibilities with regard to individual students. The instructor believed that some students in the class viewed his role as a facilitator in the student's own construction of knowledge rather than a dispenser of knowledge as an abrogation of the teacher's traditional duty, to help students when they do not understand and become confused. He tried to solve this difficulty by questioning the student to ascertain just what the student understood about the problem and what solution he or she had devised, then would give the student just enough direction to make the student's time spent on the problem profitable. One of the rules of the class was that the instructor would not comment upon the correctness of the answer or method until the particular problem had been thoroughly discussed by the class as a whole. After the problem had been discussed by the larger group, the instructor was more open to helping individual students understand and solve the problem.

The greatest weakness of using a constructivist approach to instruction in this class was that it made it more difficult to cover all the material in the course syllabus and to touch upon all the information in the textbook. The advantage of the traditional lecture approach is its speed and efficiency. A lecturer can cover vast quantities of material in a single class, providing the correct method of doing problems, elegant demonstrations or polished proofs, and an encyclopedia of information. In contrast to the traditional approach, a constructivist approach is often ponderously slow in covering the material. The instructor must be content to focus on key concepts, to allow students enough time to talk about their ideas, and to realize that not all information will be touched upon. He or she must also recognize that students will devise solutions which may be correct yet are inefficient, awkward, and time–consuming. The presentation of some of these solutions to the class as a whole and the subsequent attempts to explain the rationale behind the solutions takes time. The instructor concluded this class with the belief that it had been a more productive experience for the students, providing them with a better understanding of the content and better skills for being mathematics teachers, but he still had the unsettling feeling that he had not touched upon everything he had in previous lecture classes.

STUDENT EVALUATION OF INSTRUCTION

Near the end of the class the students each wrote an open–ended evaluation of the instructional technique used in the class. They were asked to comment upon its effectiveness when compared with other instructional techniques used in their college or university classes. They were requested to be honest and not to concern themselves with the effectiveness of the instructor, the grading policies, the course material, etc. These evaluations were anonymous and were collected together at one time in the class.

Each response was categorized by the instructor as VERY POSITIVE, NEUTRAL, or VERY NEGATIVE. Although it had been requested both in writing and verbally before the evaluation, the instructor tried to ascertain as carefully as possible each student's reaction to the instructional technique used in the class and not to other emotional factors such as testing or his own personality. A summary of the results follows:

VERY POSITIVE	NEUTRAL	VERY NEGATIVE
64%	19%	17%

The above results were not viewed as the ultimate criterion of the success or failure of using a constructivist approach in this class but rather as a rough measure of the general success of the method and a framework in which to interpret the detailed responses of the students.

Upon analyzing the student responses, two themes recurred throughout, first, the students mentioned having a better understanding of the concepts, algorithms, and rules than in previous mathematics courses, and, second, they had a better attitude toward mathematics as a result of this class. They claimed to have less fear, anxiety, and stress, and more confidence. The responses were reread and results were tabulated. Fifty percent of the class (n=18) reported having a better understanding of the material of this class than in previous classes, and thirty–six percent (n=13) reported having a better attitude toward mathematics and taking math courses. The students had not been requested to comment upon either of these areas in the evaluation.

The unfavorable responses were then carefully examined. The students seemed to express the greatest dissatisfaction with the frustration and confusion brought on by having to collaborate with their peers in solving a problem rather than seeing a correct solution immediately when they could not solve the problem. A typical response was, "I enjoyed working in groups but, when we were all lost, there was limited instruction by the instructor. To me, I need to see problems demonstrated over and over again on the board." The method of instruction also caused discomfort to some students because there were sometimes different solutions proposed by different groups to a given problem. These students were deeply disturbed by the fact that the instructor did not give the class one correct, "real" way to do a problem, but allowed them to use their own ways, provided they were mathematically sound. The instructor did guide the discussion along lines that seemed to be directed toward the construction of correct mathematical principles, and tried not to allow wrong answers, faulty procedures, or illogical reasoning to pass without comment. Another aspect about the class that disturbed some students was that they did not cover more territory. They believed that they had learned more in other classes because they had homework problems every night and a quiz on these problems the next day in an established routine. As a result of this, they believed that they had learned more mathematics and learned it better. Another concern voiced by some students was that they found that some members of their group were not pulling their weight and were sliding by. Along the same lines, some students were troubled by the composition of their group and suggested changing groups throughout the class. In retrospect, this seemed like a good idea. This was also a valid concern, since it seemed that the nontraditional students, often a very motivated and mature subset of the class, banded together quickly, while a number of groups in the class appeared to have mostly students with a rather weak mathematical background.

On the positive side, many students commented that they understood the material better than in previous classes. A response typical of many others was, "In this type of class the students are encouraged to explore and really learn, rather than simply memorizing a long list of trivial facts. The pace and format of this class allow me to really get a handle on math." Another student remarked that the method used in this class "helped me understand the way a problem really works, not just how to plug numbers into a meaningless formula." There were many other similar comments.

An aspect of the instructional method frequently mentioned was that it sanctioned different approaches to a problem and different interpretations, making many students more relaxed and less anxious about trying to comprehend the teacher's way of doing a problem. As one student mentioned, when talking to the instructor during the class, "Sometimes I just have to close my ears and not listen to the teacher, because, if I do, I lose all my own understanding of the problem and become totally confused." Once again, one student's response typifies many others, "For the first time in my life, I have found math to be an enjoyable and 'doable' subject. As with many people, math frustrates and scares me. I hate all the red marks on homework, and I hate that there is (or is thought to be) ONE (student's capitalization) way to work a math problem, and only ONE absolute answer for everything. I have really enjoyed this instructional technique. It is motivational and creates a pleasant nonthreatening environment that seems to be helpful (to me) in learning math concepts." Many other students shared the above student's enjoyment of the class. One frequent comment from students was that they had been bored and lost during the lectures in other classes and liked being active rather than passive during the class period. A number of students also mentioned that in math classes in the past, when they had become confused, they were too frightened to ask a question in front of the whole class. As one student admitted, "in my Theory I lecture course I never even listened. I just took notes. I did this because I was constantly confused and too scared to interrupt him and be singled out. The group work was very helpful because I could learn from them as well as from you... I felt free to ask any question at any time."

For many students, especially those who had been very successful in other math classes, this class was a shock. It took some time for them to get accustomed to the method. As one student honestly revealed, "I feel constructivism is a great way to teach. When I get a classroom of my own I plan on using it... There is only one problem I have with it... To switch gears like this is very disconcerting and frustrating, when I have been trained with a system so long... For me it has been like a new pair of underwear. At first they are very uncomfortable but pretty soon they become a part of you."

ASSESSMENT OF BASIC PROBABILITY CONCEPTS

An instrument was devised by the instructor consisting of various problems to assess the students' understanding of basic probability concepts. Problems were designed which demanded as little specific knowledge of probability as possible yet whose solution would illustrate the understanding of a particular concept. Problems to be a specific part of the course curriculum were avoided. The instrument was administered at the beginning and end of the course with the results compiled for each administration. An answer to a problem was categorized as correct if either the student used a common method for solving the problem which demonstrated understanding of the concept or used their own method, provided it was mathematically sound. Except in the most obvious cases, the students were asked to explain carefully how they had attempted to solve each problem. The results of this assessment are tabulated below:

CONCEPT	PRETEST UNDERSTANDING	POSTTEST UNDERSTANDING
Sample space	19%	69%
Empirical probability	29%	60%
Fundamental counting principle	63%	77%
Asymmetric probability tree diagrams	0%	11%
Symmetric tree diagrams	3%	69%
Simple properties of probability	47%	84%
Permutations	3%	51%
Combinations	0%	43%
Conditional probability	29%	34%
Expected value	0%	11%

The idea of a sample space, a list of the possible outcomes of a probability experiment whose categories are mutually exclusive and exhaustive, is a critical concept for the study of elementary probability. A key organizing principle for elementary probability study is to find a sample space for an experiment, then determine the probabilities associated with each of the desired outcomes (the successes) or events and calculate the final answer. Tree diagrams can be an important tool for systematically finding all the outcomes of a sample space and determining their respective probabilities. Likewise, permutations and combinations are more sophisticated tools for probability study, especially in calculating the number of elements in a sample space. Most of the students in the pretest had no idea that it was important to try to list all the possible outcomes of a probability experiment, instead solving the problems by adding or multiplying the most obvious numbers. Throughout the class it became obvious that some students had difficulty with creating an organized and systematic representation of all the possible outcomes. This inability to formulate a clear scheme for organizing and categorizing became especially transparent when the topic was permutations and combinations. According to Piaget and Inhelder (Piaget & Inhelder, 1975), the ability to understand both permutations and combinations demands the use of formal operations. They describe how children progress through three stages in understanding of combinations: empirical combinations, searching for a system, and discovery of a system, and in permutations: absence of a system, the empirical discovery of partial systems, and the discovery of a system. Although these students are not children, some seemed to be on each of the three levels for both subjects. One group of students, when called upon to share their solution to the problem of finding all groups of five objects taken three at a time, began listing their results in an apparently haphazard and random manner. Questioned by the instructor about their system, they replied that they had listed every group they could think of until they began duplicating those on the list. Likewise, another group, when attempting to solve the problem of determining the probability of having exactly three girls in a family of 4 children, created a list of thirteen different arrangements in a similar random manner, thinking of as many arrangements as they could until they could not think of any more.

In the pretest students could not solve probability problems which might be considered routine by using a simple tree diagram to organize the outcomes. One of the more interesting problems in the class was the following: Suppose team A has won 2/3 of its games against team B during the regular season. What is the probability that it will win a best two–out–of–three postseason playoff? Every group answering the question responded that it was 2 out of 3 times. The answer was trivial. There were the numbers 2 out of 3 appearing twice in the problem, so it must be 2/3. Besides, the students reasoned that if team A had won 2 out of 3 games during the season, then they would win 2 out of 3 games in the playoff and win the playoff. This problem intrigued the class and, by its counterintuitive nature, convinced some of the class of the importance of a careful examination of the sample space. As the above results show, the students of this class had great difficulty in solving problems which might be modelled by an asymmetric tree diagram. This was especially true if dependent events were involved (irreplaceable cards or balls) and the probabilities changed throughout the problem.

There was at times in the class a blurred distinction between empirical and theoretical probability. The instructor assumed that the students' solutions to problems could sometimes be verified or refuted by using empirical (hands–on) probability or simulations, but there was some confusion about this. When the class sought to validate their solutions to the problem of finding the probability of throwing a sum of 7 or 11 on a pair of dice, there was a period of disagreement while groups defended their positions, until one student explained that both answers appearing on the blackboard were correct, the one in theory and the other by observation. Another instance of the confusion between empirical and theoretical probability came with the problem: If you have a die known to be fair, yet have tossed 10 consecutive heads, what is the probability of throwing a heads on the next toss? Many students were unconvinced by the idea that the probability was 1/2, arguing that a tails would be more likely since heads had appeared so often, or that, since heads had appeared so many times in a row, it was more likely. There were many quizzical looks when the instructor explained that getting 10 heads in a row did not affect the probability on the next throw. Borovcnik and Bentz have a detailed discussion of this problem (Borovcnik & Bentz, 1991).

It should be noted that the topics of conditional probability and expected value were only treated briefly in the class, which might explain the lack of understanding shown by the students on the posttest. Students had little understanding of either concept coming into the class, although the assessment problems were straight forward. This did not surprise the instructor, since many students in this level of class are not able to compute a weighted average, a concept very similar to expected value.

CONCLUSION

If the mathematics education community is truly interested in implementing the recommendations proposed in the NCTM's Standards, teaching mathematics courses for preservice elementary teachers at the college or university in a more progressive manner would make a contribution. The constructivist approach used in this class did a better job of preparing the students to become elementary mathematics teachers than the traditional format used in previous classes by the instructor and was more consonant with the ideas expressed in the Standards. In this classroom problem solving was the central focus of the curriculum, the primary goal of all instruction, and an integral part of all mathematical activity, as the Standards recommend. Students worked cooperatively in small groups or in whole class discussions in solving problem. They were doing mathematics themselves rather than watching the professor do mathematics. They were given the opportunity to read, write, and discuss ideas in order to better communicate mathematically and the opportunity to reason mathematically by making conjectures, gathering evidence, and building arguments.

In this study the students gained an improved understanding of some of the key concepts, procedures, and reasoning of elementary probability theory. This study was not designed to be an experimental study comparing the academic achievement of students taught with a problem solving approach with those taught by a lecture approach, but was an exploratory venture by one instructor to test the viability of using the former method to teach elementary probability. By the end of this class the instructor became convinced that a constructivist approach coupled with problem solving brought about many positive changes in the students, a better ability to communicate about mathematics, more confidence in facing a large group and explaining mathematics, better skills of knowing how to solve a problem and how to gain information about it, an improved capability to write about mathematics, a better attitude toward the subject, and an improved understanding of probability. There was a tradeoff in changing instructional methods. It was impossible to cover as much material and for the students to be exposed to as much information. In the future, as with constructivism in the elementary schools, large–scale experimental studies are needed to determine if preservice elementary teachers, taught with a constructivist approach, have indeed gained better understanding in mathematics and have experienced the positive affective changes mentioned above. The majority of students in this class preferred the new approach. This fact alone encourages further research in this area.

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